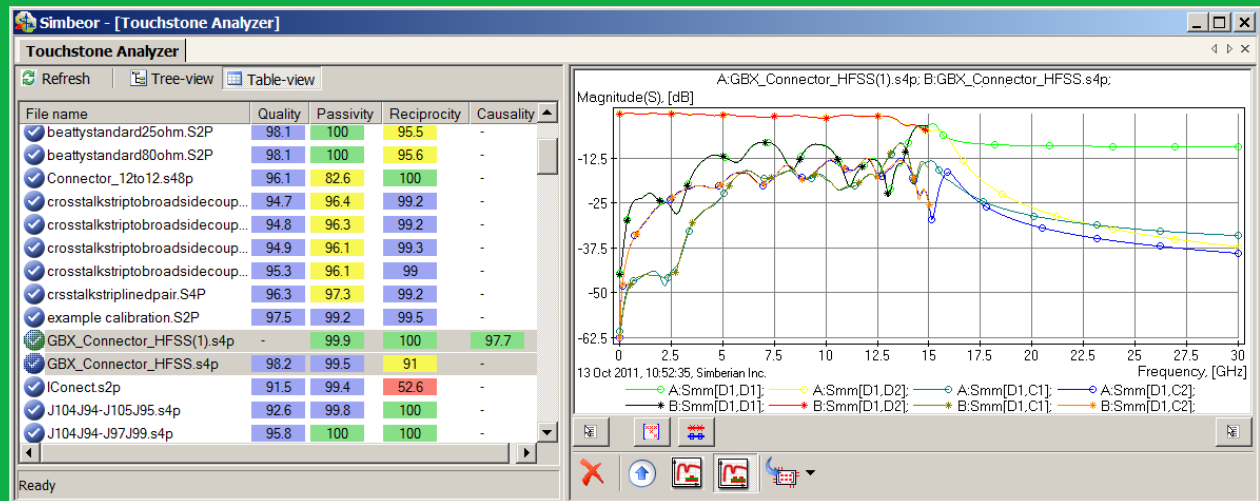


# S-Parameter Quality Metrics

Yuriy Shlepnev

Simberian Inc.

[www.simberian.com](http://www.simberian.com)



ANRITSU Signal Integrity Symposium, Santa Clara and Newport Beach, CA, July 2014

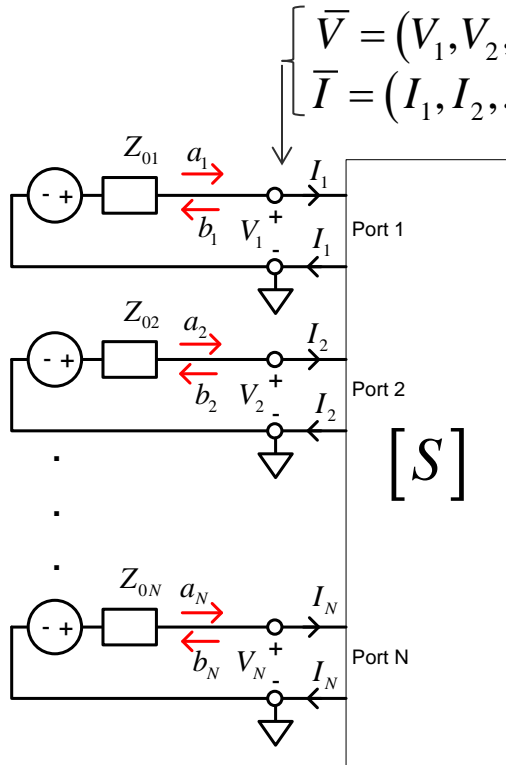
# Outline

- Introduction
- S-parameters in frequency and time domains
- Constrains on S-parameters
- Quality metrics for reciprocity, passivity, causality
- Rational approximation and final quality metric
- Simbeor Touchstone Analyzer
- Conclusion
- Contacts and resources

# Introduction

- S-parameter models are becoming ubiquitous in design of multi-gigabit interconnects
  - Connectors, cables, PCBs, packages, backplanes, ... ,any LTI-system in general can be characterized with S-parameters from DC to daylight
- Electromagnetic or circuit analysis or measurements with VNA or TDNA are used to build S-parameter models mostly in Touchstone form (discrete, band-limited)
- Very often such models have quality issues:
  - Passivity and causality violations
  - Reciprocity violations
  - Common sense violations
- **And produce different time-domain and even frequency-domain responses in different solvers!**
- This session covers some basics of S-parameter model quality evaluation and improvement for interconnect analysis

# Multiport S-parameters Definition



$$\begin{cases} \bar{V} = (V_1, V_2, \dots, V_N)^t & \text{- vector of port voltages} \\ \bar{I} = (I_1, I_2, \dots, I_N)^t & \text{- vector of port currents} \end{cases}$$

$$Z_0 = \text{diag}\{Z_{0i}, i = 1, \dots, N\} \in \mathbb{C}^{N \times N} \quad \text{normalization impedances}$$

$$\bar{a} = \frac{1}{2} Z_0^{-1/2} \cdot (\bar{V} + Z_0 \cdot \bar{I}) \quad \text{- vector of incident waves}$$

$$\bar{b} = \frac{1}{2} Z_0^{-1/2} \cdot (\bar{V} - Z_0 \cdot \bar{I}) \quad \text{- vector of reflected waves}$$

Scattering matrix definition (**Frequency Domain**):

$$\bar{b} = S \cdot \bar{a}, \quad S \in \mathbb{C}^{N \times N}, \quad S_{i,j} = \left. \frac{b_i}{a_j} \right|_{a_k=0 \ k \neq j}$$

Reduces a system description to a simple input-output relationship irrespective of internal structure!

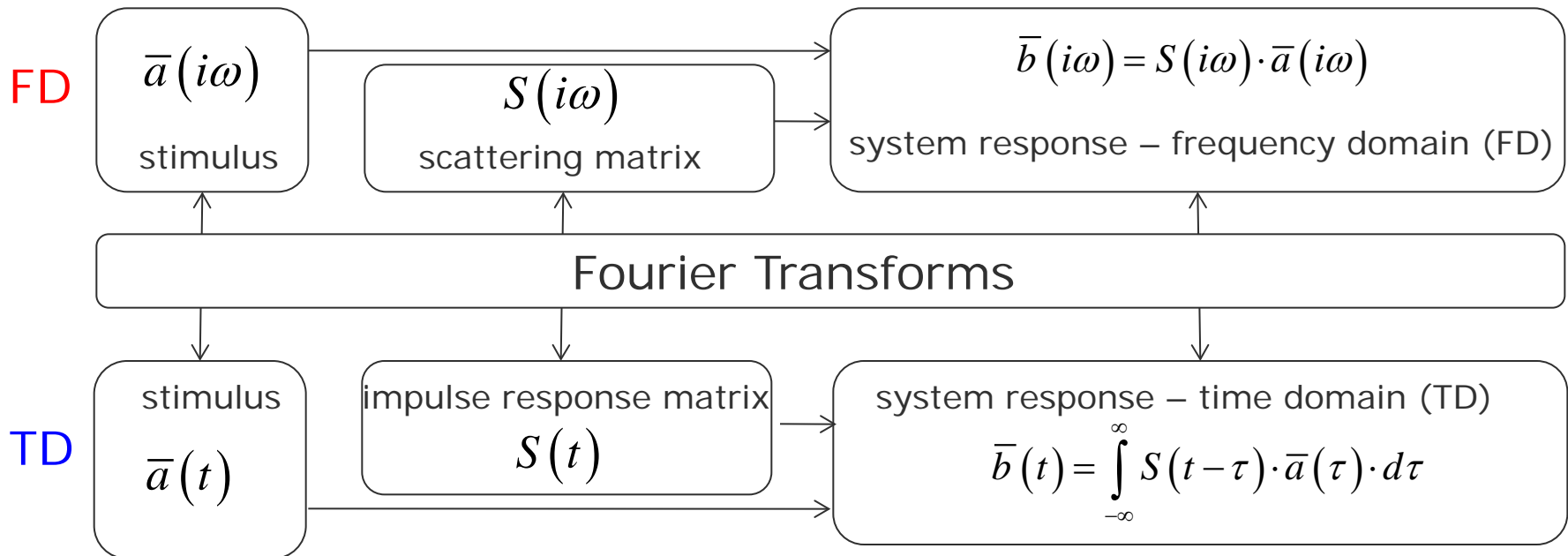
Reflected wave at port  $i$  with unit incident wave at port  $j$  defines scattering parameter  $S[i,j]$

More in D.M. Pozar, Microwave engineering, John Wiley & Sons, 1998.

# System Response Computation Requires Frequency-Continuous S-parameters from DC to Infinity

Frequency domain is preferable for analysis of interconnects

$$S(i\omega) = \int_{-\infty}^{\infty} S(t) \cdot e^{-i\omega t} \cdot dt, \quad S(i\omega) \in \mathbb{C}^{N \times N}$$



Time domain analysis may be also needed!

$$S(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(i\omega) \cdot e^{i\omega t} \cdot d\omega, \quad S(t) \in \mathbb{R}^{N \times N}$$

# Possible Approximations for Discrete Models

- Discrete Fourier Transform (DFT) and convolution
  - Slow and may require interpolation and extrapolation of tabulated S-parameters (uncontrollable error)
  - See more on typical problems with DFT in  
P. Pupalaiakis, “The Relationship Between Discrete-Frequency S-Parameters and Continuous-Frequency Responses”, DesignCon, Santa Clara CA, 2012
- Approximate discrete S-parameters with frequency-continuous rational functions (controllable error)
  - Accuracy control over defined frequency band (RMS error)
  - Causal functions (with passivity enforcement) defined from DC to infinity with analytical impulse response
  - Fast recursive convolution algorithm to compute TD response
  - Results consistent in time and frequency domains
- **Not all Touchstone models are suitable for either approach**

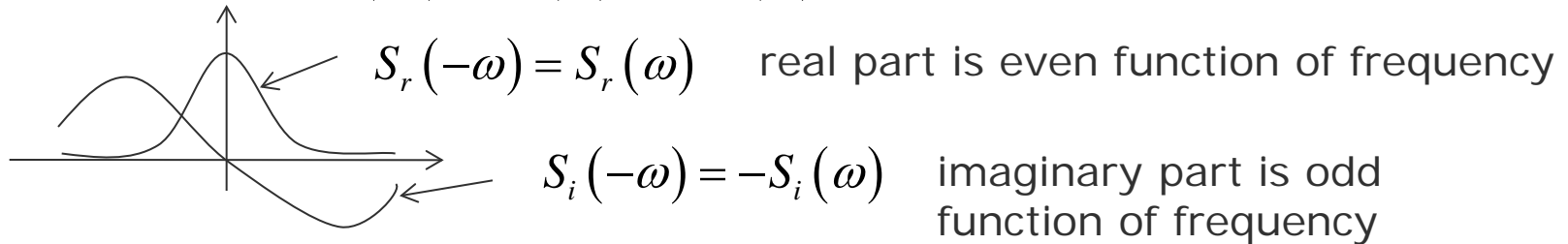
What are the constraints on S-parameters?

# Realness Constraint on Time-Domain Response

- Time-domain impulse response matrix must be real function of time

$$S(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(i\omega) \cdot e^{i\omega t} \cdot d\omega, \quad S(t) \in \mathbb{R}^{N \times N}$$

- It is true if  $S(i\omega) = S_r(\omega) + i \cdot S_i(\omega)$  and



- Those conditions are satisfied by default because of we do not use negative frequencies in Touchstone models
- Conditions at zero frequency are useful to restore the DC point:

$$\left. \frac{dS_r(\omega)}{d\omega} \right|_{\omega=0} = 0, \quad S_i(0) = 0$$

DC condition for all multiport parameters

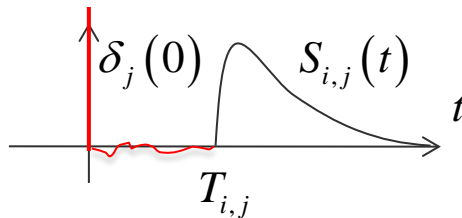




# Causality Estimation - Difficult Way

- Kramers-Kronig relations **cannot be directly used to verify causality** for the frequency-domain response known over **the limited bandwidth at some points**
- Causality boundaries can be introduced to estimate causality of the tabulated and band-limited data sets
  - Milton, G.W., Eyre, D.J. and Mantese, J.V, *Finite Frequency Range Kramers Kronig Relations: Bounds on the Dispersion*, Phys. Rev. Lett. 79, 1997, p. 3062-3064
  - Triverio, P. Grivet-Talocia S., *Robust Causality Characterization via Generalized Dispersion Relations*, IEEE Trans. on Adv. Packaging, N 3, 2008, p. 579-593.

**Even if test passes – a lot of uncertainties due to band limitedness and discreteness**



**Band limitedness of FD response**  
**Multipath propagation**

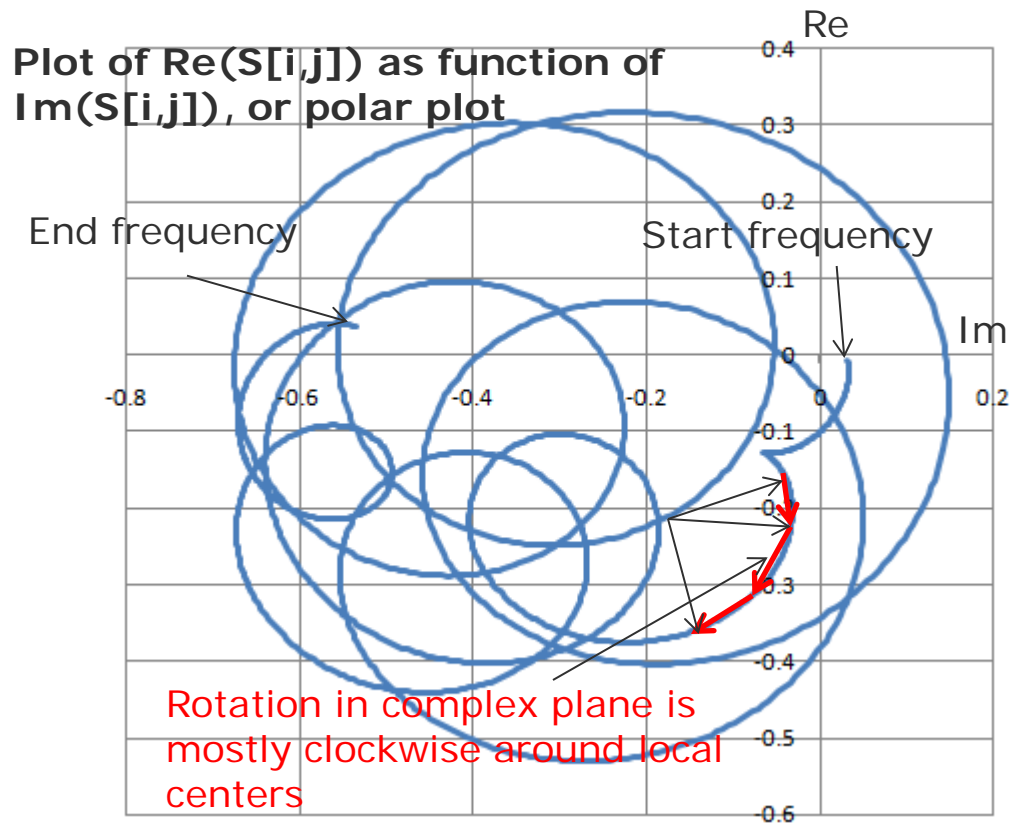
**Superluminality:** Q. Zhang, et al., Wave-Interference Explanation of Group-Delay Dispersion in Resonators, IEEE Antennas and Propagation Magazine, 2013, v. 55, N2, p. 212-227.

**Temporal leakage:** A.R. Djordjevic et al., Temporal Leakage in Analysis of Electromagnetic Systems, IEEE Antennas and Propagation Magazine, v. 54, N6, 2012, p. 92 - 101.



# “Causality” Estimation - Easy Way

- “Heuristic” causality measure based on the observation that polar plot of a causal system rotates mostly clockwise (suggested by V. Dmitriev-Zdorov)



Causality measure (CM) can be computed as the ratio of clockwise rotation measure to total rotation measure in %.

If this value is below 80%, the parameters are reported as suspect for possible violation of causality.

Algorithm is good for numerical models (to find under-sampling), but no so good for measured data due to noise!

# Passivity and Causality in Time-Domain

- A multiport network is passive if energy absorbed by multiport

$$E(t) = \int_{-\infty}^t [\bar{a}^t(\tau) \cdot \bar{a}(\tau) - \bar{b}^t(\tau) \cdot \bar{b}(\tau)] \cdot d\tau \geq 0, \quad \forall t$$

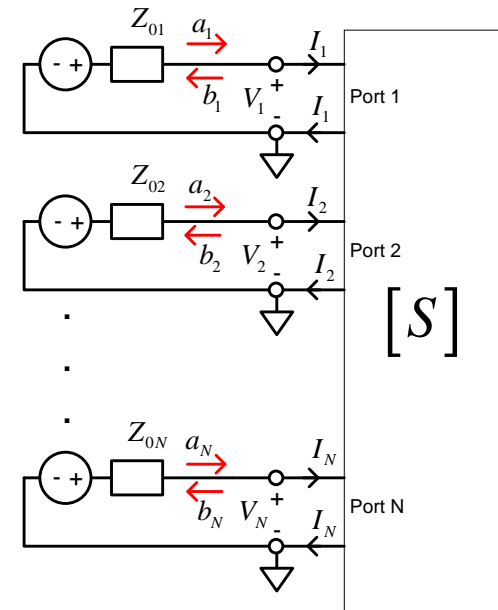
(does not generate energy)

for all possible incident waves

- If the system is passive according to the above definition, it is also causal

$$\bar{a}(t) = 0, \quad \forall t < t_0 \Rightarrow \int_{-\infty}^t [\bar{b}^t(\tau) \cdot \bar{b}(\tau)] \cdot d\tau \leq 0 \Rightarrow \bar{b}(t) = 0, \quad \forall t < t_0$$

- Thus, we need to check only the passivity of interconnect model!



More in: P. Triverio S. Grivet-Talocia, M.S. Nakhla, F.G. Canavero, R. Achar, Stability, Causality, and Passivity in Electrical Interconnect Models, IEEE Trans. on Advanced Packaging, vol. 30. 2007, N4, p. 795-808.

# Passivity in Frequency Domain

- Power transmitted to multiport is a difference of power transmitted by incident and reflected waves:

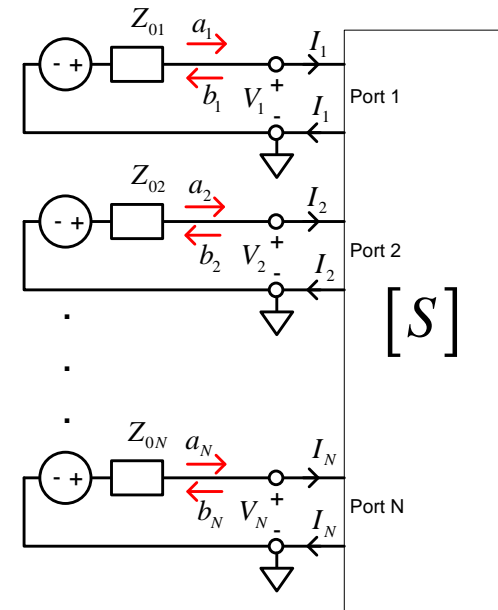
$$P_{in} = \sum_{n=1}^N |a_n|^2 - |b_n|^2 = [\bar{a}^* \cdot \bar{a} - \bar{b}^* \cdot \bar{b}]$$

or 
$$P_{in} = \bar{a}^* \cdot \bar{a} - \bar{a}^* \cdot S^* S \cdot \bar{a} = \bar{a}^* \cdot [U - S^* S] \cdot \bar{a}$$

- Transmitted power is defined by Hermitian quadratic form and must be not negative for passive multiport for any combination of incident waves
- Quadratic form is non-negative if eigenvalues of the matrix are non-negative (Golub & Van Loan):

$$\text{eigenvals}[U - S^* \cdot S] \geq 0 \quad \Rightarrow \quad \text{eigenvals}[S^* \cdot S] \leq 1 \quad (U \text{ is unit matrix})$$

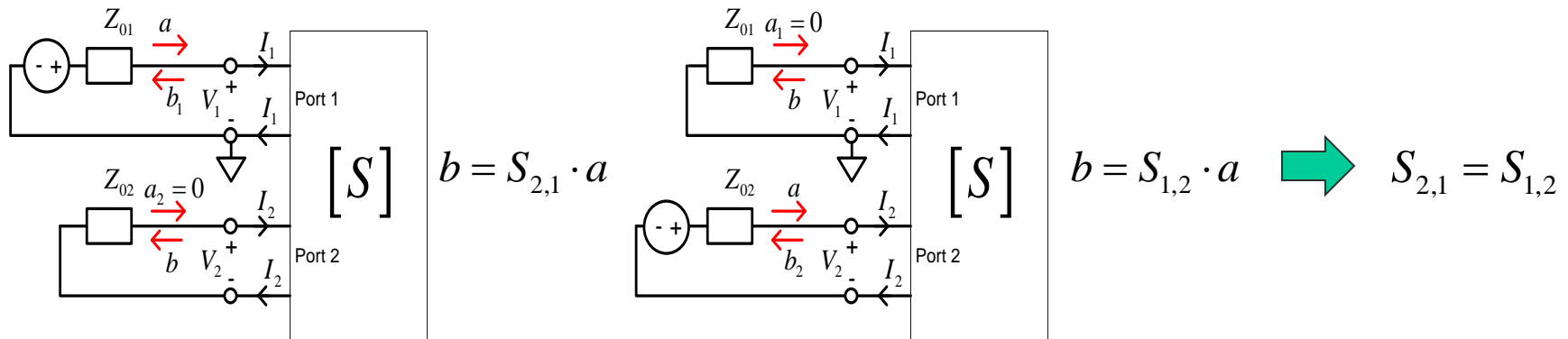
*Sufficient condition only if verified from DC to infinity  
(impossible for discrete Touchstone models)*



# Reciprocity

- Linear circuits with reciprocal materials are reciprocal according to **Lorentz's theorem of reciprocity**:

Reflected wave measured at port 2 with incident wave at port 1 is equal to reflected wave measured at port 1 with the same incident wave at port 2



- In general it means that **the scattering matrices are symmetric**

$$S_{i,j} = S_{j,i} \text{ or } S = S^t \quad \text{at all frequencies}$$

More in: L. Sevgi "Reciprocity: Some Remarks from a Field Point of View", IEEE Antennas and Propagation Magazine, Vol. 52, No.2, April 2010

# Good S-parameter Models of Interconnects

- Must be passive (do not generate energy)

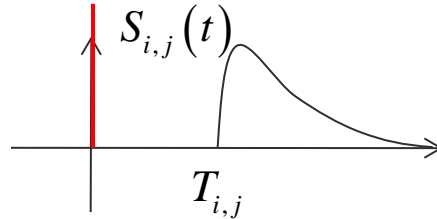
$$P_{in} = \bar{a}^* \cdot [U - S^* S] \cdot \bar{a} \geq 0 \quad \Rightarrow \quad \text{eigenvals}[S^* \cdot S] \leq 1 \quad \text{from DC to infinity!}$$

- Must be reciprocal (linear reciprocal materials used in PCBs)

$$S_{i,j} = S_{j,i} \quad \text{or} \quad S = S^t$$

- Must be causal (have causal step or impulse response or satisfy KK relations)

$$S_{i,j}(t) = 0, \quad t < T_{ij}$$



$$S(i\omega) = \frac{1}{i\pi} PV \int_{-\infty}^{\infty} \frac{S(i\omega')}{\omega - \omega'} \cdot d\omega'$$

- Must have sufficient bandwidth matching signal spectrum
- Must be appropriately sampled to resolve all resonances

# Quality Metrics (0-100%) to Define Goodness

First introduced at IBIS forum at DesignCon 2010

- Passivity Quality Measure:

$$PQM = \max \left[ \frac{100}{N_{total}} \left( N_{total} - \sum_{n=1}^{N_{total}} PW_n \right), 0 \right] \% \quad PW_n = 0 \text{ if } PM_n < 1.00001; \text{ otherwise } PW_n = \frac{PM_n - 1.00001}{0.1}$$

should be >99%

$$PM_n = \sqrt{\max \left[ \text{eigenvals} \left( S^*(f_n) \cdot S(f_n) \right) \right]}$$

- Reciprocity Quality Measure:

$$RQM = \max \left[ \frac{100}{N_{total}} \left( N_{total} - \sum_{n=1}^{N_{total}} RW_n \right), 0 \right] \% \quad RW_n = 0 \text{ if } RM_n < 10^{-6}; \text{ otherwise } RW_n = \frac{RM_n - 10^{-6}}{0.1}$$

should be >99%

$$RM_n = \frac{1}{N_s} \sum_{i,j} |S_{i,j}(f_n) - S_{j,i}(f_n)|$$

- Causality Quality Measure: Minimal ratio of clockwise rotation measure to total rotation measure in % (should be >80% for numerical models)

# Preliminary Quality Estimation Metrics

- Preliminary Touchstone model quality can be estimated with Passivity, Reciprocity and Causality quality metrics (PQM, RQM, CQM)

Metric/Model Icon	✔ - good	✔ - acceptable	⚠ - inconclusive	✖ - bad
Passivity	[100, 99.9]	(99.9, 99]	(99, 80]	(80, 0]
Reciprocity	[100, 99.9]	(99.9, 99]	(99, 80]	(80, 0]
Causality	[100, 80]	(80, 50]	(50, 0]	-----

Color code	Passivity (PQM)	Reciprocity (RQM)	Causality (CQM)
Green – good	[99.9, 100]	[99.9, 100]	[80, 100]
Blue – acceptable	[99, 99.9)	[99, 99.9)	[50, 80)
Yellow – inconclusive	[80, 99)	[80, 99)	[20, 50)
Red - bad	[0, 80)	[0, 80)	[0, 20)



# Example of Preliminary Quality Estimation in Simbeor Touchstone Analyzer™

Small passivity & reciprocity violations in most of the models  
 Low causality in some measured data due to noise at high frequencies



# Rational Approximation of S-parameters as Alternative Frequency-Continuous Model

$$\bar{b} = S \cdot \bar{a}, \quad S_{i,j} = \frac{b_i}{a_j} \Big|_{a_k=0, k \neq j} \Rightarrow S_{i,j}(i\omega) = \left[ d_{ij} + \sum_{n=1}^{N_{ij}} \left( \frac{r_{ij,n}}{i\omega - p_{ij,n}} + \frac{r_{ij,n}^*}{i\omega - p_{ij,n}^*} \right) \right] \cdot e^{-s \cdot T_{ij}}$$

$s = i\omega$ ,  $d_{ij}$  – values at  $\infty$ ,  $N_{ij}$  – number of poles,

$r_{ij,n}$  – residues,  $p_{ij,n}$  – poles (real or complex),  $T_{ij}$  – optional delay

Continuous functions of frequency defined from DC to infinity;

Causal if passivity is ensured!

- Impulse response is analytical, real and delay-causal:

$$S_{i,j}(t) = 0, \quad t < T_{ij}$$

$$S_{i,j}(t) = d_{ij} \delta(t - T_{ij}) + \sum_{n=1}^{N_{ij}} \left[ r_{ij,n} \cdot \exp(p_{ij,n} \cdot (t - T_{ij})) + r_{ij,n}^* \cdot \exp(p_{ij,n}^* \cdot (t - T_{ij})) \right], \quad t \geq T_{ij}$$

- Stable  $\text{Re}(p_{ij,n}) < 0$

- Passive if  $\text{eigenvals} [S(\omega) \cdot S^*(\omega)] \leq 1 \quad \forall \omega, \text{ from } 0 \text{ to } \infty$

- Reciprocal if  $S_{i,j}(\omega) = S_{j,i}(\omega)$

May require enforcement

# Uses for Rational Approximation

- Compute time-domain response of a channel with a fast recursive convolution algorithm (exact solution for PWL signals)
- Produce broad-band SPICE macro-models
  - Smaller model size, stable analysis
  - Consistent frequency and time domain analyses in any solver
- Improve quality of tabulated Touchstone models
  - Fix minor passivity and causality violations
  - Interpolate and extrapolate with guaranteed passivity and causality
- **Measure the original model quality**

# Quality Estimation with Rational Model

- Accuracy of discrete S-parameters approximation with frequency-continuous macro-model, passive from DC to infinity

$$RMSE = \max_{i,j} \left[ \sqrt{\frac{1}{N} \sum_{n=1}^N |S_{ij}(n) - S_{ij}(\omega_n)|^2} \right]$$

*original tabulated data*
 $S_{i,j}(i\omega) = \left[ d_{ij} + \sum_{n=1}^{N_{ij}} \left( \frac{r_{ij,n}}{i\omega - p_{ij,n}} + \frac{r_{ij,n}^*}{i\omega - p_{ij,n}^*} \right) \right] \cdot e^{-sT_{ij}}$

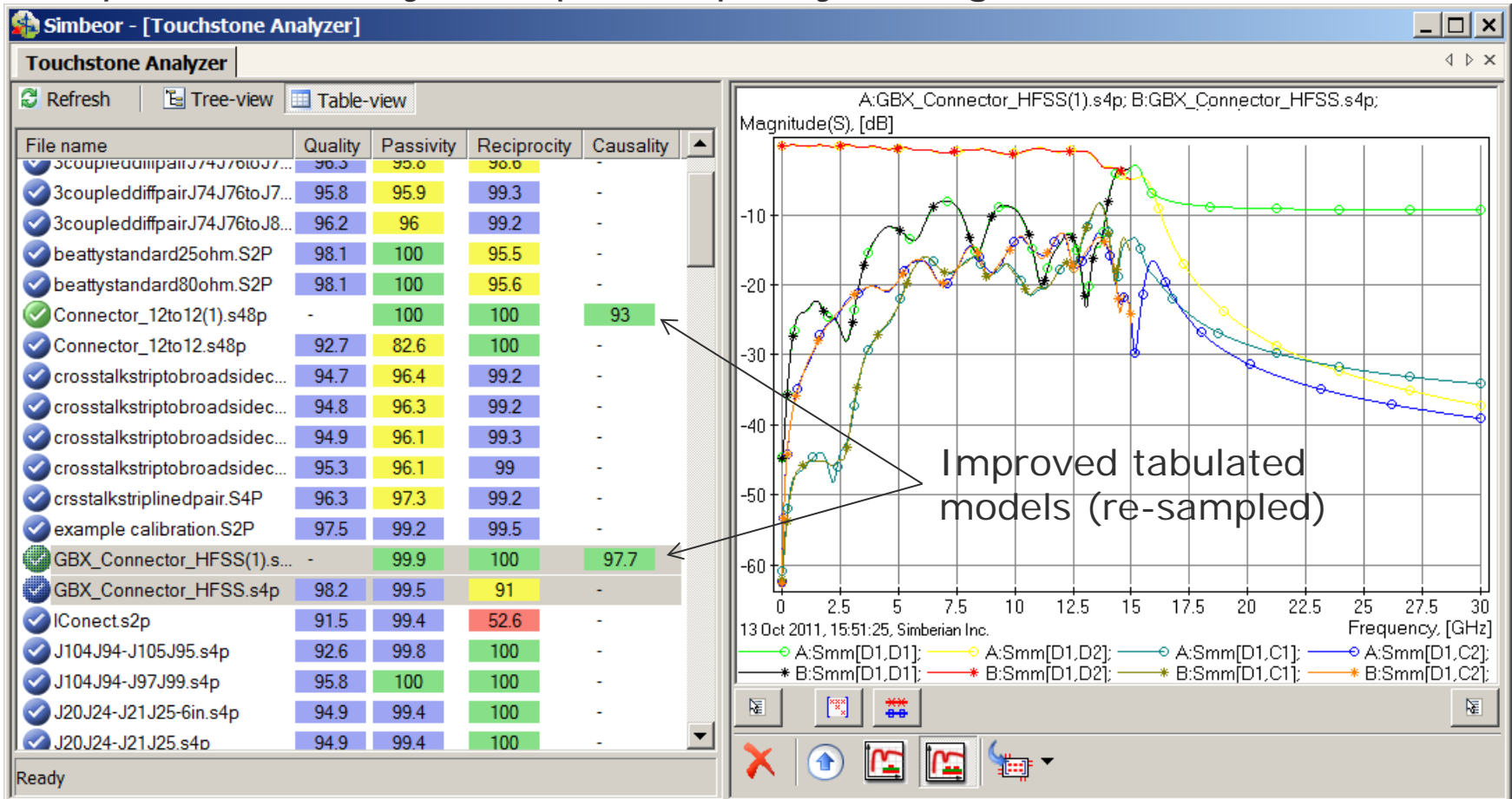
- Can be used to estimate quality of the original data

$$Q = 100 \cdot \max(1 - RMSE, 0) \%$$

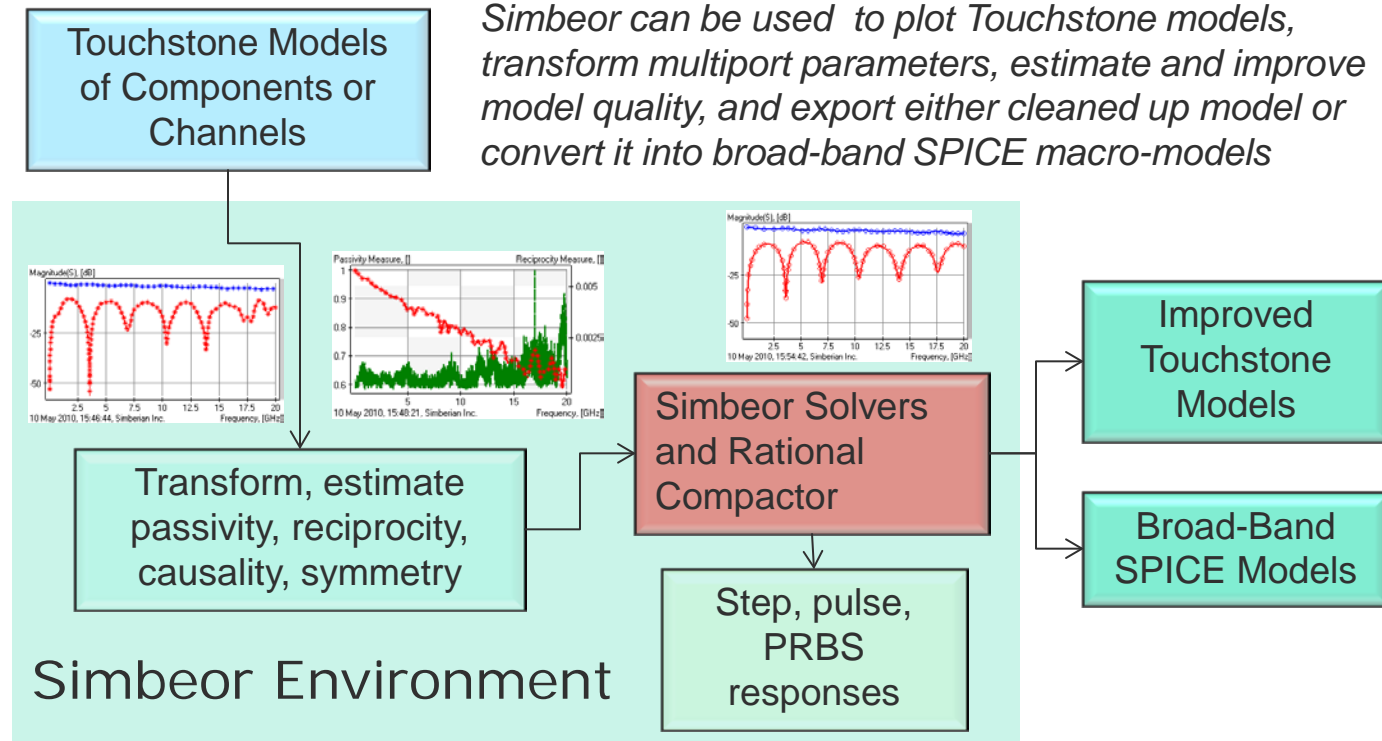
Model Icon/Quality	Quality Metric	RMSE
✔ - good	[99, 100]	[0, 0.01]
✓ - acceptable	[90, 99)	(0.01, 0.1]
? - inconclusive	[50, 90)	(0.1, 0.5]
✖ - bad	[0, 50)	> 0.5
❓ - uncertain	[0,100], not passive or not reciprocal	

# Example of Quality Estimation with RCM in Simbeor Touchstone Analyzer®

All rational macro-models are passive, reciprocal, causal and have acceptable accuracy (acceptable quality of original models)



# Simbeor Touchstone Analyzer for Model Quality Assurance, Clean-up and Macro-modeling



Simbeor Touchstone Analyzer™ facilitates and automates all quality assurance and macro-modeling tasks

# Demo: Simbeor Touchstone Analyzer™

- Find all Touchstone models in computer or in the network and estimates passivity, reciprocity and causality
- Plot S-parameters and quality and compliance metrics
- Build macro-model and use it for final quality estimation
- Produce BB SPICE or improved Touchstone models
- Import model into a project for further analysis or use in a linear network

# Conclusion & Questions



# How to Avoid Problems with S-parameter Models?

- Use reciprocity, passivity and causality metrics for preliminary analysis
  - RQM and PQM metrics should be > 99% (acceptable level)
  - CQM should be > 80% for all numerical models
- Use the rational model accuracy as the final quality measure
  - QM should be > 90% (acceptable level)
- **Discard the model with low RQM, PQM and QM metrics!**
  - The main reason is we do not know what it should be
- Models that pass the quality metrics may still be not usable or mishandled by a system simulator
  - **Due to band-limitedness, discreteness and brut force model fixing**
- Use rational or BB SPICE macro-models instead of Touchstone models for consistent time and frequency domain analyses

# Contact & Resources

- Yuriy Shlepnev, Simberian Inc.  
[shlepnev@simberian.com](mailto:shlepnev@simberian.com)  
Tel: 206-409-2368
- Download Simbeor® from [www.simberian.com](http://www.simberian.com) and try Touchstone Analyzer™ on your models and all other features for 15 days
- To learn more on S-parameters quality see the following presentations (also available at Simberian web site and on request):
  - Y. Shlepnev, Quality Metrics for S-parameter Models, DesignCon 2010 IBIS Summit, Santa Clara, February 4, 2010
  - H. Barnes, Y. Shlepnev, J. Nadolny, T. Dagostino, S. McMorrow, Quality of High Frequency Measurements: Practical Examples, Theoretical Foundations, and Successful Techniques that Work Past the 40GHz Realm, DesignCon 2010, Santa Clara, February 1, 2010.
  - E. Bogatin, B. Kirk, M. Jenkins, Y. Shlepnev, M. Steinberger, How to Avoid Butchering S-Parameters, DesignCon 2011
  - Y. Shlepnev, Reflections on S-parameter quality, DesignCon 2011 IBIS Summit, Santa Clara, February 3, 2011



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