Conductor surface roughness modeling:
From “snowballs” to “cannonballs”

Yuriy Shlepnev, Simberian Inc.
March 18, 2019, revised on March 21 and 28, 2019

Huray “snowball” or “cannonball” conductor surface roughness model – which one to use? This short paper explains/demystifies different versions of the Huray model that was originally introduced in [1].

“Snowball” models. Paper [1] uses approximate expression for power absorbed by a single electrically small sphere, to derive the first order approximation for absorption by a set of spheres using simple superposition of absorptions. Equation (9) in the paper [1] provides approximate expression for ratio of the power absorbed by a set of independent spheres to power absorbed by flat surface as follows:

\[ K_H = \frac{P_{\text{rough}}}{P_{\text{smooth}}} = 1 + 3 \sum_{i=1}^{N_r} \frac{4\pi r_i^2 A_{\text{hex}}}{A_{\text{hex}}} \left(1 + \frac{\delta}{r_i} + \frac{\delta^2}{2r_i^2}\right)^{-1} \]  (1)

Were \( \delta = (\pi f \mu \sigma)^{-1/2} \) is skin depth (frequency-dependent parameter), \( N_r \) is the number of spheres or “snowballs” with radius \( r_i \) and \( A_{\text{hex}} \) is the area of the flat surface on which the spheres would be stacked. \( K_H \) characterizes the increase in the conductor losses comparing to flat surface – it can be used as the roughness correction coefficient in the analysis of PCB or packaging interconnects [2] - [4]. The maximal value of the loss increase is given by equation (8) in [1] as follows

\[ K_{H_{\text{max}}} = 1 + 3 \sum_{i=1}^{N_r} \frac{4\pi r_i^2 A_{\text{hex}}}{A_{\text{hex}}} \]  (2)

As we can see from the formula (2) and as stated in the paper [1] the maximal power loss depends “only on the area of all of the “snowballs” or “anchor nodules” on a hexagonal area relative to the flat copper surface area upon which they sit. The power lost in this model is independent of RMS deviation of the snowball stack up height or the RMS surface roughness!” This is the consequence of the assumption that all spheres are actually not interacting – it is just the first order approximation. The model does not describe any particular arrangements of the spheres – it is just superposition of the absorptions by multiple independent spheres. This is really important to understand, to avoid confusions and falling into the trap of geometrical assumptions - the model (1) should be used as a phenomenological one. The hexagonal shape of the area and sphere stacking was used as a way to compute minimal and maximal number of spheres, to evaluate reasonable number of snowballs in the model. As it is stated on page 15 “The hexagonal geometry choice has no bearing on the final power loss calculation but it gives us a basis to find the lower and upper limits for average size snowballs that fill the area and look something like the SEM photographs in figure 10 and 11.” It does not give boundaries for possible losses either. Because of that, all attempts to get parameters in formula (1) from the geometrical assumptions are groundless – the copper surface geometry is not described by the model and the model is too approximate for the actual surfaces. Though, because of it captures the physics of absorption increase with the frequency by
a set of spheres, it can be used as the fitting formula. It similar to the way we use Debye model for
dielectrics for instance – we do not associate it with a particular geometry of dipoles formed by atoms or
molecules, but use it as the phenomenological formula with parameters fitted for different materials.
Same is applicable to formula (1) – it captures the physics of the abortion by a set of small spheres.
Spheres with different radii are needed for more flexibility in the model (this is similar to multi-pole
Debye models for dielectrics). There are three parameters in the original formula (1) - \( N_i \), \( A_{\text{hex}} \) and \( r_i \),
but only 2 are independent. Parameters in the maximal possible loss increase (2) can be united into one
parameter \( RF_i \) per one ball radius as follows:

\[
RF_i = 1 + \frac{\frac{3}{2} N_i \cdot 4\pi r_i^2}{A_{\text{hex}}} \quad (3)
\]

That parameter is called roughness factor in Simbeor software and characterizes maximal possible
increase of the losses for a set of “snowballs” with one radius \( r_i \).

With definition (3) expression (1) becomes much simpler:

\[
K_{\mu} = 1 + \sum_i \left( RF_i - 1 \right) \left( 1 + \frac{\delta}{r_i} + \frac{\delta^2}{2r_i^2} \right)^{-1} \quad (4)
\]

This is exactly the same model as described by (1). \( RF_i \) defined by (3) is the first parameter and ball
radius is the second parameter (\( SR_i \)) in the unified roughness model in Simbeor software [4].

Another option to rewrite equation (1) with the smaller number of parameters is as follows

\[
K_{\mu} = 1 + \frac{3}{2} \sum_i s_{r_i} \left( 1 + \frac{\delta}{r_i} + \frac{\delta^2}{2r_i^2} \right)^{-1} \quad (5)
\]

Parameter \( s_{r_i} \) is called Hall-Huray surface ratio (it is ratio of all ball surface area to a flat area) and used
in HFSS software (for one-ball model). Again, formula (5) is exactly the same as (1) or (4). The relation of
\( s_{r_i} \) with \( RF_i \) is very simple [4]:

\[
RF_i = 1 + \frac{3}{2} s_{r_i} ; \quad s_{r_i} = \frac{2}{3} \cdot (RF_i - 1) \quad (6)
\]

Note, that formulas (1), (4) and (5) are all equivalent and can be called Huray “snowball” model. It is
one-ball (or one-level in Simbeor) model when just one ball size is used. With multiple ball radii it is
multi-ball or multi-level model as defined in the unified model in Simbeor software. The frequency-
dependent part of the Huray “snowball” model was further modified in [5], to have surface impedance
boundary conditions for flat conductor causal. With the modification suggested in [5], formula (4)
becomes as follows

\[
K_{\mu} = 1 + \sum_i \left( RF_i - 1 \right) \left( 1 + \left( 1 - j \right) \frac{\delta}{2r_i} \right)^{-1} \quad (7)
\]
This roughness correction coefficient is complex, but the real part of surface impedance with complex correction coefficient (7) is exactly equal to the real part of surface impedance defined with real equation (4). Complex coefficient (7) increases the conductor internal inductance, to preserve causality. It is called Huray-Bracken model in Simbeor software [4] (it is multi-level). Expression (5) can be also rewritten in the causality-preserved form (7) that, apparently, is used in HFSS software.

“Cannonball” models. Huray “snowball” model defined by (4) has 2 parameters per level – \(RF_i\) (or \(sr_i\) in (5)) and ball radius \(r_i\). Where to get those parameters? There were some scientific attempts of doing it [6] without much success at this point. The idea is to identify surface features that will define the skin-effect onset and associate them with sphere radii and identify surface area increase that will define the maximal loss increase per level (roughness factor). Both parameters cannot be found from simple mechanical measurements of \(Ra\) (arithmetical mean), \(Rq\) (RMS value) or \(Rz\) (average distance between highest peaks and lowers valleys) – such parameters do not have information on the shape of the surface. Without the information on the shape, it is not possible to predict RF. Though, the “cannonball” model introduced in [7]-[10] promises that with just \(Ra\) or \(Rz\) parameters available from some manufacturers the model can predict the conductor losses. Every time the author uses \(Ra\) or \(Rz\), he gets excellent correlation in the losses – the method works like a magic in all examples. Let’s take a closer look at the “cannonball” model. As was stated in the original paper [1] the geometrical arrangement of the balls on hexagonal surface was purely for evaluation of the reasonable number of spheres – the result does not correlate with RMS or other mechanical description of the rough surface in general. Though, the “snowball” arrangement was taken too literally and 11 spheres on hexagonal surface were used in [7] to obtain the following equation (follows from equation (11) and (15) in [7])

\[
K_{c1} = 1 + \frac{11\sqrt{3} \pi}{4 + 2\sqrt{3}} \left(1 + \frac{\delta}{r} + \frac{\delta^2}{2r^2}\right)^{-1}
\]

This is basically just one-ball Huray “snowball” model (4) with the roughness factor RF (3) fixed to

\[
RF_{c1} = 1 + \frac{11\sqrt{3} \pi}{4 + 2\sqrt{3}} \approx 9
\]

This value is actually within the reasonable range for some types of copper – see examples in [12] and [13]. Note that RF in model (8)-(9) is independent of \(Ra\) or \(Rz\). Considering the “cannonball” radius, author of [7] assumes that the rough surface has triangular profile with 60 degrees angle at the top and evaluates height \(H_{rms}\) of the cannonball stacks with either \(Ra\) or \(Rz\) (equations (12) and (13) in [7]) and use \(H_{rms}\) to evaluate the radius of one ball (equation (14) in [7]) as follows

\[
r_{c1} = \frac{H_{rms}}{2\left(\frac{2\sqrt{6}}{3} + 1\right)}
\]

Where
\[ H_{rms} = \frac{R_z}{2\sqrt{3}} \quad \text{or} \quad H_{rms} = \frac{R_a}{2\sqrt{3}} \]  \hspace{1cm} (11)

*Hrms* here should not be confused with the Rq or RMS value measured for rough surfaces. There are no conversion formulas between Ra, Rq and Rz in general. Also, use of Ra or Rz is confusing – these parameters can be different by order of magnitude [11]! Though, the author recommends use of Rz for the matte side and Ra for the drum side of copper foil [7]. It is confusing already. In summary, the “cannonball” model (8) is just one-parameter version of the Huray “snowball” model (4) with RF parameter fixed to 9. The only parameter is the sphere radius that can be computed from either Ra or Rz with (10). All we can state is that some value of radius computed within this range may actually provide the loss values close to the measured. Though, it can be done only if the losses are known in advance from measurements.

Another version of the “cannonball” model is suggested and used in [8]-[9]. 11 spheres on a square surface were used to “derive” two parameters for the original Huray “snowball” model. From equation (2) and (4) in [8] the correction coefficient of this model can be expressed as follows

\[ K_{c2} = 1 + \frac{7 \cdot \pi}{3} \left(1 + \frac{\delta}{r} + \frac{\delta^2}{2r^2}\right)^{-1} \]  \hspace{1cm} (12)

Again, this is just one-parameter Huray “snowball” model (4) with the roughness factor (3) fixed to \[ RF_{c2} = 1 + \frac{7 \cdot \pi}{3} \approx 8.33 \]  \hspace{1cm} (13)

Again, RF=8.33 is within the range of reasonable values for some types of copper [12], [13]. In addition, the model defines sphere radius as follows:

\[ r_{c2} = \frac{H_{max}}{2(1+\sqrt{2})} \]  \hspace{1cm} (14)

Where \( H_{rms} \) is defined similar to the hexagonal model (11) with either Ra or Rz. In the practical examples of [8] and [9], Rz was used to show excellent correlation. However, Rz on the treated side of copper was “guessed” and the roughness correction coefficient was averaged for the drum and matter sides, that is technically equivalent of use of 2-ball Huray model with the fixed roughness factor and two ball radii as the parameters defined as follows:

\[ K_{c3} = 1 + \frac{7 \cdot \pi}{6} \sum_{i=1}^{2} \left(1 + \frac{\delta}{r_i} + \frac{\delta^2}{2r_i^2}\right)^{-1} \]  \hspace{1cm} (15)

Model (15) is the two-ball Huray “snowball” model with the roughness factor parameters fixed for each level to the following value:

\[ RF_{c3} = 1 + \frac{7 \cdot \pi}{6} \approx 4.665 \]  \hspace{1cm} (16)
This is “cannonball” model #3 – which one is in your EDA tool?

**Conclusion:** Huray “snowball” model has 2 independent parameters per one level. It captures the physics of the loss increase on the rough surface. Direct derivation of the parameters from the geometry of rough surface may be possible, but it will definitely take more than just Ra or Rz. The only viable option at this point is the fitting the model to either measured GMS-parameters or complex propagation constant (Gamma), as it is done in [12] and [13].

All “cannonball” models are just Huray “snowball” models with the fixed roughness factors and with either one or two ball radii as the parameters. There is nothing wrong with the model, but it is not more accurate than the original “snowball” model – the fixation of parameters cannot make it more accurate, it just makes it less flexible and, potentially, less accurate. Any reasonable guess for the ball radius would provide increase of the conductor losses and impression that it works. Use of Ra or Rz to derive the ball radius is pure guess that may create the impression that model works with the spreadsheet parameters for the copper. To see if it works in your case, select either one-level (8) or (12) or two-level model (15). Fix the RF parameters to values provided by (9), (13) or (16). Take either Ra or Rz from the copper foil spreadsheet (if you find one) and define the ball radius (radii) with (11), (10) or (14). Is this better than nothing? – it depends on how good your guess is. In reality, you will simply end up with a wide range of possible outcomes and start wondering which one will work. The best way to validate which one is correct is to compare the model with the measured GMS-parameters or Gamma. Without the measured data the model would be not reliable, to say the least, and cannot be used in real design. With the measurements available, no guesses is needed – the radius (or radii) can be simply adjusted to fit to the measured data as it is done in [12] and [13] (see more references on the model identification in the papers). If the fitting with the acceptable accuracy is not possible, use the original “snowball” model with 2 parameters (2-parameter model is more flexible). If accurate fitting is still not possible, increase the number of levels to two – that will give very flexible model with 4 parameters.

**Revision:** The author of the “cannonball” model clarified what is the “cannonball” model at SI List [14]. The true "cannonball" model is the one-ball Huray model (4) with the roughness factor (RF) fixed to 8.33 (or Hall-Huray surface ratio fixed to 4.887) and the radius of the spheres set to \( r = 0.06 \times R_z \) (follows from formula (11) with Rz and (14) or Equation 11 in [15]). It is defined by equation (12) with \( r = 0.06 \times R_z \). Now it is even simpler to set it in Simbeor - just find “proper” \( R_z \) and set RF=8.33 and \( SR = 0.06 \times R_z \) – as simple as that! The author also explains [14] how he has got and “adjusted” \( R_z \) for CMP-28 case used in multiple papers with the excellent correlation to measurements. Note that in every case the measurements were available in advance and one parameter was sufficient to tune it. Otherwise the guess of the single parameter would be as good as nothing and \( R_z \) has nothing to do with that.
References

5. J. E. Bracken, A causal Huray model for surface roughness, DesignCon 2012
15. L. Simonovich, PCB interconnect modeling demystified, DesignCon2019