S-Parameter Quality Metrics

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Outline

• Introduction
• S-parameters in frequency and time domains
• Constrains on S-parameters
• Quality metrics for reciprocity, passivity, causality
• Rational approximation and final quality metric
• Simbeor Touchstone Analyzer
• Conclusion
• Contacts and resources
Introduction

• S-parameter models are becoming ubiquitous in design of multi-gigabit interconnects
  – Connectors, cables, PCBs, packages, backplanes, … , any LTI-system in general can be characterized with S-parameters from DC to daylight
• Electromagnetic or circuit analysis or measurements with VNA or TDNA are used to build S-parameter models mostly in Touchstone form (discrete, band-limited)
• Very often such models have quality issues:
  – Passivity and causality violations
  – Reciprocity violations
  – Common sense violations
• And produce different time-domain and even frequency-domain responses in different solvers!
• This session covers some basics of S-parameter model quality evaluation and improvement for interconnect analysis
Multiport S-parameters Definition

Scattering matrix definition (Frequency Domain):

\[
\bar{b} = S \cdot \bar{a}, \quad S \in C^{N \times N}, \quad S_{i,j} = \frac{b_i}{a_j} \quad a_k = 0 \quad k \neq j
\]

Reflected wave at port i with unit incident wave at port j defines scattering parameter \(S[i,j]\)

System Response Computation Requires Frequency-Continuous S-parameters from DC to Infinity

Frequency domain is preferable for analysis of interconnects

\[ S(i\omega) = \int_{-\infty}^{\infty} S(t) \cdot e^{-i\omega t} \cdot dt, \quad S(i\omega) \in \mathbb{C}^{N \times N} \]

**Stimulus**

**Scattering Matrix**

**System Response – Frequency Domain (FD)**

\[ b(i\omega) = S(i\omega) \cdot \bar{a}(i\omega) \]

**Fourier Transforms**

**Stimulus**

**Impulse Response Matrix**

**System Response – Time Domain (TD)**

\[ b(t) = \int_{-\infty}^{\infty} S(t-\tau) \cdot \bar{a}(\tau) \cdot d\tau \]

**Time domain analysis may be also needed!**

\[ S(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(i\omega) \cdot e^{i\omega t} \cdot d\omega, \quad S(t) \in \mathbb{R}^{N \times N} \]
Possible Approximations for Discrete Models

• Discrete Fourier Transform (DFT) and convolution
  – Slow and may require interpolation and extrapolation of tabulated S-parameters (uncontrollable error)
  – See more on typical problems with DFT in

• Approximate discrete S-parameters with frequency-continuous rational functions (controllable error)
  – Accuracy control over defined frequency band (RMS error)
  – Causal functions (with passivity enforcement) defined from DC to infinity with analytical impulse response
  – Fast recursive convolution algorithm to compute TD response
  – Results consistent in time and frequency domains

• Not all Touchstone models are suitable for either approach

What are the constrains on S-parameters?
Realness Constrain on Time-Domain Response

- Time-domain impulse response matrix must be real function of time
  \[ S(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(i\omega) \cdot e^{i\omega t} \cdot d\omega, \quad S(t) \in R^{N \times N} \]

- It is true if \( S(i\omega) = S_r(\omega) + i \cdot S_i(\omega) \) and
  \[ S_r(-\omega) = S_r(\omega) \quad \text{real part is even function of frequency} \]
  \[ S_i(-\omega) = -S_i(\omega) \quad \text{imaginary part is odd function of frequency} \]

- Those conditions are satisfied by default because of we do not use negative frequencies in Touchstone models
- Conditions at zero frequency are useful to restore the DC point:
  \[ \left. \frac{dS_r(\omega)}{d\omega} \right|_{\omega=0} = 0, \quad S_i(0) = 0 \quad \text{DC condition for all multiport parameters} \]
Causality of LTI System (TD & FD)

• The system is causal if and only if all elements of the time-domain impulse response matrix are \( S_{i,j}(t) = 0 \) at \( t < 0 \)

delayed causality (for interconnects):

\[
S_{i,j}(t) = 0 \text{ at } t < T_{i,j}, T_{i,j} > 0
\]

• This lead to Kramers-Kronig relations in frequency-domain

\[
S(i\omega) = \frac{1}{i\pi} \text{PV} \int_{-\infty}^{\infty} \frac{S(i\omega')}{\omega - \omega'} \cdot d\omega', \quad \text{PV} = \lim_{\varepsilon \to 0} \left( \int_{-\infty}^{\omega - \varepsilon} + \int_{\omega + \varepsilon}^{+\infty} \right)
\]

\[
S_r(\omega) = \frac{1}{\pi} \text{PV} \int_{-\infty}^{\infty} \frac{S_i(\omega')}{\omega - \omega'} \cdot d\omega', \quad S_i(\omega) = -\frac{1}{\pi} \text{PV} \int_{-\infty}^{\infty} \frac{S_r(\omega')}{\omega - \omega'} \cdot d\omega'
\]

Causality Estimation - Difficult Way

- Kramers-Kronig relations cannot be directly used to verify causality for the frequency-domain response known over the limited bandwidth at some points
- Causality boundaries can be introduced to estimate causality of the tabulated and band-limited data sets

Even if test passes – a lot of uncertainties due to band limitedness and discreteness

\[ \delta_j(0), S_{i,j}(t) \]

Band limitedness of FD response
Multipath propagation


“Causality” Estimation - Easy Way

• “Heuristic” causality measure based on the observation that polar plot of a causal system rotates mostly clockwise (suggested by V. Dmitriev-Zdorov)

![Polar plot of Re(S[i,j]) as function of Im(S[i,j]), or polar plot](image)

Rotation in complex plane is mostly clockwise around local centers

Causality measure (CM) can be computed as the ratio of clockwise rotation measure to total rotation measure in %.

If this value is below 80%, the parameters are reported as suspect for possible violation of causality.

Algorithm is good for numerical models (to find under-sampling), but no so good for measured data due to noise!
Passivity and Causality in Time-Domain

• A multiport network is passive if energy absorbed by multiport

\[ E(t) = \int_{-\infty}^{t} \left[ \bar{a}'(\tau) \cdot \bar{a}(\tau) - \bar{b}'(\tau) \cdot \bar{b}(\tau) \right] \cdot d\tau \geq 0, \ \forall t \]

(does not generate energy)

for all possible incident waves

• If the system is passive according to the above definition, it is also causal

\[ \bar{a}(t) = 0, \ \forall t < t_0 \Rightarrow \int_{-\infty}^{t} \left[ \bar{b}'(\tau) \cdot \bar{b}(\tau) \right] \cdot d\tau \leq 0 \Rightarrow \bar{b}(t) = 0, \ \forall t < t_0 \]

• Thus, we need to check only the passivity of interconnect model!

Passivity in Frequency Domain

- Power transmitted to multiport is a difference of power transmitted by incident and reflected waves:
  \[ P_{in} = \sum_{n=1}^{N} |a_n|^2 - |b_n|^2 = [\bar{a}^* \cdot \bar{a} - \bar{b}^* \cdot \bar{b}] \]
  or
  \[ P_{in} = \bar{a}^* \cdot \bar{a} - \bar{a}^* \cdot S^* S \cdot \bar{a} = \bar{a}^* \cdot [U - S^* S] \cdot \bar{a} \]

- Transmitted power is defined by Hermitian quadratic form and must be not negative for passive multiport for any combination of incident waves

- Quadratic form is non-negative if eigenvalues of the matrix are non-negative (Golub & Van Loan):
  \[ \text{eigenvals} \left[ U - S^* \cdot S \right] \geq 0 \quad \Rightarrow \quad \text{eigenvals} \left[ S^* \cdot S \right] \leq 1 \quad (U \text{ is unit matrix}) \]

Sufficient condition only if verified from DC to infinity (impossible for discrete Touchstone models)
Reciprocity

• Linear circuits with reciprocal materials are reciprocal according to Lorentz’s theorem of reciprocity:
  Reflected wave measured at port 2 with incident wave at port 1 is equal to reflected wave measured at port 1 with the same incident wave at port 2

\[
S \begin{bmatrix} a_1 \rightarrow I_1 \\ b_1 \rightarrow V_1 \end{bmatrix} = S_{2,1} \cdot a
\]

\[
S \begin{bmatrix} a_2 \rightarrow I_2 \\ b_2 \rightarrow V_2 \end{bmatrix} = S_{2,1} \cdot a
\]

• In general it means that the scattering matrices are symmetric

\[
S_{i,j} = S_{j,i} \quad \text{or} \quad S = S^t
\]

at all frequencies

Good S-parameter Models of Interconnects

- Must be passive (do not generate energy)

\[ P_{in} = \bar{a}^* \cdot \left[ U - S^* S \right] \cdot \bar{a} \geq 0 \quad \Rightarrow \quad \text{eigenvals} \left[ S^* \cdot S \right] \leq 1 \quad \text{from DC to infinity!} \]

- Must be reciprocal (linear reciprocal materials used in PCBs)

\[ S_{i,j} = S_{j,i} \quad \text{or} \quad S = S^t \]

- Must be causal (have causal step or impulse response or satisfy KK relations)

\[ S_{i,j}(t) = 0, \quad t < T_{ij} \]

\[ S(i\omega) = \frac{1}{i\pi} PV \int_{-\infty}^{\infty} \frac{S(i\omega')}{\omega - \omega'} d\omega' \]

- Must have sufficient bandwidth matching signal spectrum
- Must be appropriately sampled to resolve all resonances
Quality Metrics (0-100%) to Define Goodness

First introduced at IBIS forum at DesignCon 2010

• Passivity Quality Measure:

\[ PQM = \max \left[ \frac{100}{N_{\text{total}}} \left( N_{\text{total}} - \sum_{n=1}^{N_{\text{total}}} PW_n \right), 0 \right] \% \]

\[ PW_n = 0 \text{ if } PM_n < 1.00001; \text{ otherwise } PW_n = \frac{PM_n - 1.00001}{0.1} \]

should be >99%

• Reciprocity Quality Measure:

\[ RQM = \max \left[ \frac{100}{N_{\text{total}}} \left( N_{\text{total}} - \sum_{n=1}^{N_{\text{total}}} RW_n \right), 0 \right] \% \]

\[ RW_n = 0 \text{ if } RM_n < 10^{-6}; \text{ otherwise } RW_n = \frac{RM_n - 10^{-6}}{0.1} \]

\[ RM_n = \frac{1}{N_s} \sum_{i,j} \left| S_{i,j}(f_n) - S_{j,i}(f_n) \right| \]

should be >99%

• Causality Quality Measure: Minimal ratio of clockwise rotation measure to total rotation measure in % (should be >80% for numerical models)
Preliminary Quality Estimation Metrics

- Preliminary Touchstone model quality can be estimated with Passivity, Reciprocity and Causality quality metrics (PQM, RQM, CQM)

<table>
<thead>
<tr>
<th>Metric/Model Icon</th>
<th>Passivity (PQM)</th>
<th>Reciprocity (RQM)</th>
<th>Causality (CQM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green – good</td>
<td>[99.9, 100]</td>
<td>[99.9, 100]</td>
<td>[80, 100]</td>
</tr>
<tr>
<td>Blue – acceptable</td>
<td>[99, 99.9]</td>
<td>[99, 99.9]</td>
<td>[50, 80]</td>
</tr>
<tr>
<td>Yellow – inconclusive</td>
<td>[80, 99]</td>
<td>[80, 99]</td>
<td>[20, 50]</td>
</tr>
<tr>
<td>Red - bad</td>
<td>[0, 80]</td>
<td>[0, 80]</td>
<td>[0, 20]</td>
</tr>
</tbody>
</table>
Example of Preliminary Quality Estimation in Simbeor Touchstone Analyzer™

Small passivity & reciprocity violations in most of the models
Low causality in some measured data due to noise at high frequencies
Rational Approximation of S-parameters as Alternative Frequency-Continuous Model

\[
\bar{b} = S \cdot \bar{a}, \quad S_{i,j} = \left. \frac{b_i}{a_j} \right|_{a_k=0 \ k \neq j} \Rightarrow S_{i,j}(i\omega) = \left[ d_{ij} + \sum_{n=1}^{N_{ij}} \left( \frac{r_{ij,n}}{i\omega - p_{ij,n}} + \frac{r_{ij,n}^*}{i\omega - p_{ij,n}^*} \right) \right] \cdot e^{-s \cdot T_{ij}}
\]

Continuous functions of frequency defined from DC to infinity;

Causal if passivity is ensured!

- Impulse response is analytical, real and delay-causal:
  \[
  S_{i,j}(t) = 0, \quad t < T_{ij}
  \]
  \[
  S_{i,j}(t) = d_{ij} \delta(t - T_{ij}) + \sum_{n=1}^{N_{ij}} \left[ r_{ij,n} \cdot \exp \left( p_{ij,n} \cdot (t - T_{ij}) \right) + r_{ij,n}^* \cdot \exp \left( p_{ij,n}^* \cdot (t - T_{ij}) \right) \right], \quad t \geq T_{ij}
  \]

- Stable \( \text{Re}\left( p_{ij,n} \right) < 0 \)
- Passive if \( \text{eigenvals}\left[ S(\omega) \cdot S^*(\omega) \right] \leq 1 \ \forall \omega, \text{from 0 to } \infty \)
- Reciprocal if \( S_{i,j}(\omega) = S_{j,i}(\omega) \)
Uses for Rational Approximation

- Compute time-domain response of a channel with a fast recursive convolution algorithm (exact solution for PWL signals)
- Produce broad-band SPICE macro-models
  - Smaller model size, stable analysis
  - Consistent frequency and time domain analyses in any solver
- Improve quality of tabulated Touchstone models
  - Fix minor passivity and causality violations
  - Interpolate and extrapolate with guaranteed passivity and causality
- Measure the original model quality
Quality Estimation with Rational Model

- Accuracy of discrete S-parameters approximation with frequency-continuous macro-model, passive from DC to infinity

\[
RMSE = \max_{i,j} \left[ \sqrt{\frac{1}{N} \sum_{n=1}^{N} \left| S_{ij}(n) - S_{ij}(\omega_n) \right|^2} \right]
\]

- Can be used to estimate quality of the original data

\[
Q = 100 \cdot \max \left( 1 - RMSE, 0 \right) \%
\]

<table>
<thead>
<tr>
<th>Model Icon/Quality</th>
<th>Quality Metric</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>✔ - good</td>
<td>[99, 100]</td>
<td>[0, 0.01]</td>
</tr>
<tr>
<td>✔ - acceptable</td>
<td>[90, 99)</td>
<td>(0.01, 0.1]</td>
</tr>
<tr>
<td>❓ - inconclusive</td>
<td>[50, 90)</td>
<td>(0.1, 0.5]</td>
</tr>
<tr>
<td>❌ - bad</td>
<td>[0, 50)</td>
<td>&gt; 0.5</td>
</tr>
<tr>
<td>❗ - uncertain</td>
<td>[0,100], not passive or not reciprocal</td>
<td></td>
</tr>
</tbody>
</table>
Example of Quality Estimation with RCM in Simbeor Touchstone Analyzer®

All rational macro-models are passive, reciprocal, causal and have acceptable accuracy (acceptable quality of original models)
Simbeor can be used to plot Touchstone models, transform multiport parameters, estimate and improve model quality, and export either cleaned up model or convert it into broad-band SPICE macro-models.

Simbeor Touchstone Analyzer™ facilitates and automates all quality assurance and macro-modeling tasks.
Demo: Simbeor Touchstone Analyzer™

- Find all Touchstone models in computer or in the network and estimates passivity, reciprocity and causality
- Plot S-parameters and quality and compliance metrics
- Build macro-model and use it for final quality estimation
- Produce BB SPICE or improved Touchstone models
- Import model into a project for further analysis or use in a linear network
Conclusion & Questions
How to Avoid Problems with S-parameter Models?

• Use reciprocity, passivity and causality metrics for preliminary analysis
  – RQM and PQM metrics should be > 99% (acceptable level)
  – CQM should be > 80% for all numerical models
• Use the rational model accuracy as the final quality measure
  – QM should be > 90% (acceptable level)
• Discard the model with low RQM, PQM and QM metrics!
  – The main reason is we do not know what it should be
• Models that pass the quality metrics may still be not usable or mishandled by a system simulator
  – Due to band-limitedness, discreteness and brut force model fixing
• Use rational or BB SPICE macro-models instead of Touchstone models for consistent time and frequency domain analyses
Contact & Resources

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• Download Simbeor® from www.simberian.com and try Touchstone Analyzer™ on your models and all other features for 15 days

• To learn more on S-parameters quality see the following presentations (also available at Simberian web site and on request):
  – E. Bogatin, B. Kirk, M. Jenkins, Y. Shlepnev, M. Steinberger, How to Avoid Butchering S-Parameters, DesignCon 2011