Outline

- Introduction
- S-parameters in frequency and time domains
- Constrains on S-parameters in frequency domain
- Quality metrics for reciprocity, passivity, causality
- Rational approximation and final quality metric
- Conclusion
- Contacts and resources
S-parameter models

S-parameter models are becoming ubiquitous in design of multi-gigabit interconnects

- Connectors, cables, PCBs, packages, backplanes, …, any LTI-system in general can be characterized with S-parameters from DC to daylight

Electromagnetic analysis or measurements are used to build S-parameter Touchstone models

Very often such models have quality issues:

- Reciprocity violations
- Passivity and causality violations
- Common sense violations

And produce different time-domain and even frequency-domain responses in different solvers!
Multiport S-parameters formal definition

\[
\begin{align*}
\bar{V} &= (V_1, V_2, ..., V_N)^t & &\text{vector of port voltages} \\
\bar{I} &= (I_1, I_2, ..., I_N)^t & &\text{vector of port currents}
\end{align*}
\]

\[
Z_0 = \text{diag}\{Z_{0i}, i = 1, ..., N\} \in \mathbb{C}^{N \times N} & \quad \text{normalization impedances}
\]

\[
\bar{a} = \frac{1}{2} Z_0^{-1/2} \cdot (\bar{V} + Z_0 \cdot \bar{I}) & \quad \text{vector of incident waves}
\]

\[
\bar{b} = \frac{1}{2} Z_0^{-1/2} \cdot (\bar{V} - Z_0 \cdot \bar{I}) & \quad \text{vector of reflected waves}
\]

\[
[S] = \begin{bmatrix}
Z_{01} & a_1 & I_1 \\
& a_2 & I_2 \\
& & \ddots \\
& & & Z_{0N} & a_N & I_N
\end{bmatrix}
\]

\[
\bar{b} = S \cdot \bar{a}, \quad S \in \mathbb{C}^{N \times N}, \quad S_{i,j} = \left| \begin{array}{c}
\frac{b_i}{a_j} \\
\end{array} \right|_{a_k = 0 \ k \neq j}
\]

Frequency Domain (FD)

Reflected wave at port i with unit incident wave at port j defines scattering parameter \( S[i,j] \)
Example of S-parameters definition

\[
\begin{bmatrix}
  b_1 \\
  b_2
\end{bmatrix}
= \begin{bmatrix}
  S_{1,1} & S_{1,2} \\
  S_{2,1} & S_{2,2}
\end{bmatrix}
\begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix}
\]

\[v_i^+ = \sqrt{Z_0} \cdot a_i\] voltage of incident wave

\[v_i^- = \sqrt{Z_0} \cdot b_i\] voltage of reflected wave

\[V_i = v_i^+ + v_i^-\] total voltage

\[I_i = \frac{1}{Z_0} (v_i^+ - v_i^-)\] total current

\[|S_{i,j}| = \sqrt{\text{Re} \left( S_{i,j} \right)^2 + \text{Im} \left( S_{i,j} \right)^2}\] magnitude

\[|S_{i,j}|_{dB} = 20 \cdot \log \left( |S_{i,j}| \right)\] magnitude in dB

\[PORT 1\]

- \[Z_0 \xrightarrow{a_i} v_i^+ \quad I_i \xleftarrow{b_i} \]

\[S_{1,1} = \frac{b_1}{a_1}\] for \(a_2 = 0\)

\[S_{2,1} = \frac{b_2}{a_1}\] for \(a_2 = 0\)

\[S\]

\[PORT 2\]

- \[v_1^- \xrightarrow{b_2} v_2^- \quad I_2 \xleftarrow{a_2}\]

\[P_i^+ = \left| a_i \right|^2\] power of incident wave

\[P_i^- = \left| b_i \right|^2\] power of reflected wave

\[|S_{i,1}|^2 = \frac{|b_1|^2}{|a_1|^2} = \frac{P_i^-}{P_i^+}\]

\[|S_{2,1}|^2 = \frac{|b_2|^2}{|a_1|^2} = \frac{P_2^-}{P_1^+}\]

Magnitude is limited by 1 for passive systems!

\[\angle S_{i,j} = \arctan \left( \frac{\text{Im} \left( S_{i,j} \right)}{\text{Re} \left( S_{i,j} \right)} \right)\] phase

\[i = 1, 2; \quad j = 1, 2;\]
S-parameters are available in 2 forms

- Analytical models
  - Circuit with lumped elements (rational models)
  - Distributed circuits (models with delays)
- Tabulated (discrete) Touchstone models
  - SPICE simulators
  - Microwave analysis software
  - Electromagnetic analysis software
  - Measurements (VNA or TDNA)
Common S-parameter model defects

- **Model bandwidth deficiency**
  - S-parameter models are band-limited due to limited capabilities of solvers and measurement equipment
  - Model should include DC point or allow extrapolation, and high frequencies defined by the signal spectrum

- **Model discreteness**
  - Touchstone models are matrix elements at a set of frequencies
  - Interpolation or approximation of tabulated matrix elements may be necessary both for time and frequency domain analyses

- **Model distortions due to**
  - Measurement or simulation artifacts
  - Passivity violations and local “enforcements”
  - Causality violations and “enforcements”

- Human mistakes of model developers and users

- How to rate quality of the models?
System response computation requires frequency-continuous S-parameters from DC to infinity

\[
S(i\omega) = \int_{-\infty}^{\infty} S(t) \cdot e^{-i\omega t} \cdot dt, \quad S(i\omega) \in C^{N\times N}
\]

**FD**

\[\bar{a}(i\omega)\]
stimulus

\[S(i\omega)\]
scattering matrix

\[\bar{b}(i\omega) = S(i\omega) \cdot \bar{a}(i\omega)\]
system response – frequency domain (FD)

**Fourier Transforms**

**TD**

\[\bar{a}(t)\]
stimulus

\[S(t)\]
impulse response matrix

\[\bar{b}(t) = \int_{-\infty}^{\infty} S(t-\tau) \cdot \bar{a}(\tau) \cdot d\tau\]
system response – time domain (TD)

\[
S(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(i\omega) \cdot e^{i\omega t} \cdot d\omega, \quad S(t) \in R^{N\times N}
\]
Possible approximations for discrete models

- Discrete Fourier Transform (DFT) and convolution
  - Slow and may require interpolation and extrapolation of tabulated S-parameters (uncontrollable error)

- Approximate discrete S-parameters with rational functions (RMS error)
  - Accuracy is under control over the defined frequency band
  - Frequency-continuous causal functions defined from DC to infinity with analytical impulse response
  - Fast recursive convolution algorithm to compute TD response
  - Results consistent in time and frequency domains

- Not all Touchstone models are suitable for either approach
Realness constrain on time-domain response

- Time-domain impulse response matrix must be real function of time
  \[ S(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(i\omega) \cdot e^{i\omega t} \cdot d\omega, \quad S(t) \in \mathbb{R}^{N\times N} \]

- It is true if \( S(i\omega) = S_r(\omega) + i \cdot S_i(\omega) \) and
  \[ S_r(-\omega) = S_r(\omega) \quad \text{real part is even function of frequency} \]
  \[ S_i(-\omega) = -S_i(\omega) \quad \text{imaginary part is odd function of frequency} \]

- Those conditions are satisfied by default because of we do not use negative frequencies in Touchstone models

- Conditions at zero frequency are useful to restore the DC point:
  \[ \frac{dS_r(\omega)}{d\omega} \bigg|_{\omega=0} = 0, \quad S_i(0) = 0 \]
  DC condition for all multiport parameters
Causality of LTI system (TD & FD)

- The system is causal if and only if all elements of the time-domain impulse response matrix are $S_{i,j}(t) = 0$ at $t < 0$

  delayed causality (for interconnects):

  $S_{i,j}(t) = 0$ at $t < T_{i,j}$, $T_{i,j} > 0$

- This lead to Kramers-Kronig relations in frequency-domain

  $S(i\omega) = \frac{1}{i\pi} \text{PV} \int_{-\infty}^{\infty} \frac{S(i\omega')}{\omega - \omega'} \cdot d\omega'$, $PV = \lim_{\varepsilon \to 0} \left( \int_{-\infty}^{\omega-\varepsilon} + \int_{\omega+\varepsilon}^{+\infty} \right)$

  $S_r(\omega) = \frac{1}{\pi} \text{PV} \int_{-\infty}^{\infty} \frac{S_i(\omega')}{\omega - \omega'} \cdot d\omega'$, $S_i(\omega) = \frac{-1}{\pi} \text{PV} \int_{-\infty}^{\infty} \frac{S_r(\omega')}{\omega - \omega'} \cdot d\omega'$

  derivation

  $S(t) = \text{sign}(t) \cdot S(t)$,

  $\text{sign}(t) = \begin{cases} -1, & t < 0 \\ 1, & t > 0 \end{cases}$

  $S(i\omega) = \left\{ \text{sign}(t) \right\} S(t)$

  $= \frac{1}{2\pi} \left\{ \text{sign}(t) \right\} * F\left\{ S(t) \right\}$

  $F\left\{ \text{sign}(t) \right\} = \frac{2}{i\omega}$

Causality estimation - difficult way

- Kramers-Kronig relations cannot be directly used to verify causality for the frequency-domain response known over the limited bandwidth at some points.

- Causality boundaries can be introduced to estimate causality of the tabulated and band-limited data sets:

Even if test passes – a lot of uncertainties due to band limitedness.
Causality estimation - easy way

- “Heuristic” causality measure based on the observation that polar plot of a causal system rotates mostly clockwise (suggested by V. Dmitriev-Zdorov)

Plot of $\text{Re}(S[i,j])$ as function of $\text{Im}(S[i,j])$, or polar plot

Rotation in complex plane is mostly clockwise around local centers

Causality measure (CM) can be computed as the ratio of clockwise rotation measure to total rotation measure in %.

If this value is below 80%, the parameters are reported as suspect for possible violation of causality.

Algorithm is good for numerical models (to find under-sampling), but no so good for measured data due to noise!
Stability and passivity in time-domain

- The system is stable if output is bounded for all bounded inputs
  \[ |a(t)| < K \Rightarrow |b(t)| < M, \ \forall t \]  
  (BIBO)

- A multiport network is passive if energy absorbed by multiport
  \[ E(t) = \int_{-\infty}^{t} \left[ \bar{a}^T(\tau) \cdot \bar{a}(\tau) - \bar{b}^T(\tau) \cdot \bar{b}(\tau) \right] \cdot d\tau \geq 0, \ \forall t \]  
  (does not generate energy)
  for all possible incident and reflected waves

- If the system is passive according to the above definition, it is also causal
  \[ \bar{a}(t) = 0, \ \forall t < t_0 \Rightarrow \int_{-\infty}^{t} \left[ \bar{b}^T(\tau) \cdot \bar{b}(\tau) \right] \cdot d\tau \leq 0 \Rightarrow \bar{b}(t) = 0, \ \forall t < t_0 \]

- Thus, we need to check only the passivity of interconnect system!

---

Passivity in frequency domain

- Power transmitted to multiport is a difference of power transmitted by incident and reflected waves:
  \[ P_{in} = \sum_{n=1}^{N} |a_n|^2 - |b_n|^2 = \left[ \bar{a}^* \cdot \bar{a} - \bar{b}^* \cdot \bar{b} \right] \]
  or
  \[ P_{in} = \bar{a}^* \cdot \bar{a} - \bar{a}^* \cdot S^* S \cdot \bar{a} = \bar{a}^* \cdot \left[ U - S^* S \right] \cdot \bar{a} \]

- Transmitted power is defined by Hermitian quadratic form and must be not negative for passive multiport for any combination of incident waves.

- Quadratic form is non-negative if eigenvalues of the matrix are non-negative (Golub & Van Loan):
  
  \[ \text{eigenvals} \left[ U - S^* \cdot S \right] \geq 0 \quad \text{eigenvals} \left[ S^* \cdot S \right] \leq 1 \quad (U \text{ is unit matrix}) \]

**Sufficient condition only if verified from DC to infinity (impossible for discrete Touchstone models)**
Good Touchstone models of interconnects

- Must have sufficient bandwidth matching signal spectrum
- Must be appropriately sampled to resolve all resonances
- Must be reciprocal (linear reciprocal materials used in PCBs)
  \[ S_{i,j} = S_{j,i} \text{ or } S = S^t \]
- Must be passive (do not generate energy)
  \[ P_{in} = \bar{a}^* \cdot [U - S^*S] \cdot \bar{a} \geq 0 \quad \text{eigenvals} \left[ S^* \cdot S \right] \leq 1 \]  from DC to infinity!
- Have causal step or impulse response (response only after the excitation)
  \[ S_{i,j}(t) = 0, \quad t < T_{ij} \]
Quality metrics (0-100%) to define goodness

First introduced at IBIS forum at DesignCon 2010

- **Passivity Quality Measure:**

\[
PQM = \max \left[ \frac{100}{N_{total}} \left( N_{total} - \sum_{n=1}^{N_{total}} PW_n \right) , 0 \right] \%
\]

- **Reciprocity Quality Measure:**

\[
RQM = \max \left[ \frac{100}{N_{total}} \left( N_{total} - \sum_{n=1}^{N_{total}} RW_n \right) , 0 \right] 
\]

- **Causality Quality Measure:** Minimal ratio of clockwise rotation measure to total rotation measure in % (should be >80% for numerical models)

\[
CQM = \frac{1}{N_s} \sum_{i,j} \left| S_{i,j}(f_n) - S_{j,i}(f_n) \right|
\]

\[
PM_n = \sqrt{\max \left[ \text{eigenvals} \left( S^*(f_n) \cdot S(f_n) \right) \right]}
\]

\[
PW_n = 0 \text{ if } PM_n < 1.00001; \text{ otherwise } PW_n = \frac{PM_n - 1.00001}{0.1}
\]

\[
RM_n = \frac{1 - 10^{-6}}{0.1}
\]

\[
RM_n = \frac{1}{N_s} \sum_{i,j} \left| S_{i,j}(f_n) - S_{j,i}(f_n) \right|
\]
Preliminary Touchstone model quality can be estimated with Passivity, Reciprocity and Causality quality metrics (PQM, RQM, CQM)

<table>
<thead>
<tr>
<th>Metric/Model Icon</th>
<th>✔️ - good</th>
<th>✔️ - acceptable</th>
<th>❓ - inconclusive</th>
<th>🚨 - bad</th>
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<tbody>
<tr>
<td>Passivity</td>
<td>[100, 99.9]</td>
<td>(99.9, 99]</td>
<td>(99, 80]</td>
<td>(80, 0]</td>
</tr>
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<td>Reciprocity</td>
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<td>(99.9, 99]</td>
<td>(99, 80]</td>
<td>(80, 0]</td>
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<td>Causality</td>
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<td>(80, 50]</td>
<td>(50, 0]</td>
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Color code

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<tr>
<th></th>
<th>Passivity (PQM)</th>
<th>Reciprocity (RQM)</th>
<th>Causality (CQM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green – good</td>
<td>[99.9, 100]</td>
<td>[99.9, 100]</td>
<td>[80, 100]</td>
</tr>
<tr>
<td>Blue – acceptable</td>
<td>[99, 99.9]</td>
<td>[99, 99.9]</td>
<td>[50, 80]</td>
</tr>
<tr>
<td>Yellow – inconclusive</td>
<td>[80, 99]</td>
<td>[80, 99]</td>
<td>[20, 50]</td>
</tr>
<tr>
<td>Red - bad</td>
<td>[0, 80]</td>
<td>[0, 80]</td>
<td>[0, 20]</td>
</tr>
</tbody>
</table>
Example of preliminary quality estimation in Simbeor Touchstone Analyzer™

Small passivity & reciprocity violations in most of the models
Low causality in some measured data due to noise at high frequencies
Rational approximation of S-parameters as the frequency-continuous model

\[
\bar{b} = S \cdot \bar{a}, \quad S_{i,j} = \left. \frac{b_i}{a_j} \right|_{a_k = 0 \text{ } k \neq j} \Rightarrow S_{i,j}(i\omega) = \left[ d_{ij} + \sum_{n=1}^{N_{ij}} \left( \frac{r_{ij,n}}{i\omega - p_{ij,n}} + \frac{r_{ij,n}^*}{i\omega - p_{ij,n}^*} \right) \right] e^{-s \cdot T_{ij}}
\]

\( s = i\omega, \; d_{ij} - \text{values at } \infty, \; N_{ij} - \text{number of poles}, \)
\( r_{ij,n} - \text{residues}, \; p_{ij,n} - \text{poles (real or complex)}, T_{ij} - \text{optional delay} \)

- Pulse response is analytical, real and delay-causal:
  \[
  S_{i,j}(t) = 0, \quad t < T_{ij}
  \]
  \[
  S_{i,j}(t) = d_{ij} \delta(t - T_{ij}) + \sum_{n=1}^{N_{ij}} \left[ r_{ij,n} \cdot \exp\left( p_{ij,n} \cdot (t - T_{ij}) \right) + r_{ij,n}^* \cdot \exp\left( p_{ij,n}^* \cdot (t - T_{ij}) \right) \right], \quad t \geq T_{ij}
  \]

- Stable \quad \text{Re}\left( p_{ij,n} \right) < 0

- Passive if \quad \text{eigenvals}\left[ S(\omega) \cdot S^*(\omega) \right] \leq 1 \quad \forall \omega, \text{ from } 0 \text{ to } \infty \quad \text{May require enforcement}

- Reciprocal if \quad S_{i,j}(\omega) = S_{j,i}(\omega)
Bandwidth and sampling for rational approximation

- If no DC point, the lowest frequency in the sweep should be:
  - Below the transition to skin-effect (1-50 MHz for PCB applications)
  - Below the first possible resonance in the system (important for cables, L is physical length)

\[ L < \frac{\lambda}{4} = \frac{c}{4f_l \cdot \sqrt{\varepsilon_{\text{eff}}}} \]

\[ f_l < \frac{c}{4L \cdot \sqrt{\varepsilon_{\text{eff}}}} \]

- The highest frequency in the sweep must be defined by the required resolution in time-domain or by spectrum of the signal (by rise time or data rate)

\[ f_h > \frac{1}{2t_r} \]

- The sampling is very important for DFT and convolution-based algorithms, but not so for algorithms based on fitting:
  - There must be 4-5 frequency point per each resonance
  - The electrical length of a system should not change more than quarter of wave-length between two consecutive points

\[ df < \frac{c}{4L \cdot \sqrt{\varepsilon_{\text{eff}}}} \]
Rational approximation can be used to

- Compute time-domain response of a channel with a fast recursive convolution algorithm (exact solution for PWL signals)
- Improve quality of tabulated Touchstone models
  - Fix minor passivity and causality violations
  - Interpolate and extrapolate with guaranteed passivity
- Produce broad-band SPICE macro-models
  - Smaller model size, stable analysis
  - Consistent frequency and time domain analyses in any solver
- Measure the original model quality
Final quality estimation

- Accuracy of discrete S-parameters approximation with frequency-continuous macro-model, passive from DC to infinity

\[ RMSE = \max_{i,j} \left[ \sqrt{\frac{1}{N} \sum_{n=1}^{N} \left| S_{ij}(n) - S_{ij}(\omega_n) \right|^2} \right] \]

- Can be used to estimate quality of the original data

\[ Q = 100 \cdot \max \left( 1 - RMSE, 0 \right) \% \]

<table>
<thead>
<tr>
<th>Model Icon/Quality</th>
<th>Quality Metric</th>
<th>RMSE</th>
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<td>[99, 100]</td>
<td>[0, 0.01]</td>
</tr>
<tr>
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<td>(0.01, 0.1]</td>
</tr>
<tr>
<td>❓ - inconclusive</td>
<td>[50, 90)</td>
<td>(0.1, 0.5]</td>
</tr>
<tr>
<td>🚫 - bad</td>
<td>[0, 50)</td>
<td>&gt; 0.5</td>
</tr>
<tr>
<td>🚫 - uncertain</td>
<td>[0,100], not passive or not reciprocal</td>
<td></td>
</tr>
</tbody>
</table>
Example of final quality estimation in Simbeor Touchstone Analyzer®

All rational macro-models are passive, reciprocal, causal and have acceptable accuracy (acceptable quality of original models)
Conclusion: How to avoid problems with S-parameter models?

- Use reciprocity, passivity and causality metrics for preliminary analysis
  - RQM and PQM metrics should be > 99% (acceptable level)
  - CQM should be > 80% for all causal numerical models
- Use the rational model accuracy as the final quality measure
  - QM should be > 90% (acceptable level)
- Discard the model with low RQM, PQM and QM metrics!
  - The main reason is we do not know what it should be
- Models that pass the quality metrics may still be not usable or mishandled by a system simulator
  - Due to band-limitedness, discreteness and brut force model fixing
- Use rational or BB SPICE macro-models instead of Touchstone models for consistent time and frequency domain analyses
Contact and resources

- Yuriy Shlepnev, Simberian Inc.
  shlepnev@simberian.com
  Tel: 206-409-2368

- To learn more on S-parameters quality see the following presentations (also available on request):
  - Y. Shlepnev, Quality Metrics for S-parameter Models, DesignCon 2010 IBIS Summit, Santa Clara, February 4, 2010
  - E. Bogatin, B. Kirk, M. Jenkins, Y. Shlepnev, M. Steinberger, How to Avoid Butchering S-Parameters, DesignCon 2011