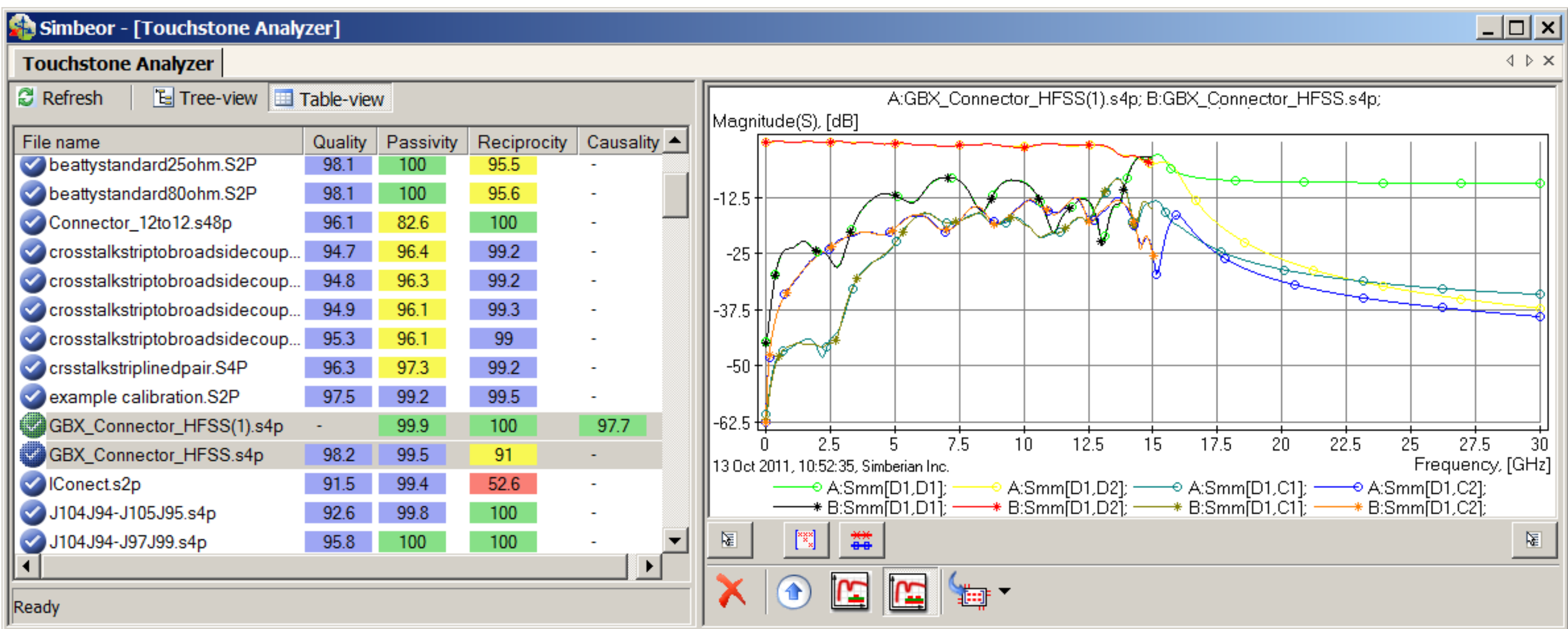
Simberian
Electromagnetic Solutions

Quality of S-parameter models

Asian IBIS Summit, Yokohama, November 18, 2011

Yuriy Shlepnev
shlepnev@simberian.com



Outline

- Introduction
- S-parameters in frequency and time domains
- Constrains on S-parameters in frequency domain
- Quality metrics for reciprocity, passivity, causality
- Rational approximation and final quality metric
- Conclusion
- Contacts and resources

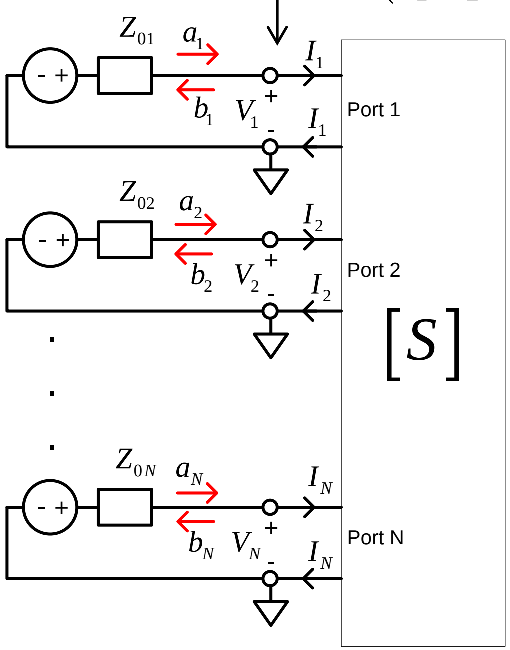
S-parameter models

- S-parameter models are becoming ubiquitous in design of multi-gigabit interconnects
  - Connectors, cables, PCBs, packages, backplanes, … ,any LTI-system in general can be characterized with S-parameters from DC to daylight
- Electromagnetic analysis or measurements are used to build S-parameter Touchstone models
- Very often such models have quality issues:
  - Reciprocity violations
  - Passivity and causality violations
  - Common sense violations
- And produce different time-domain and even frequency-domain responses in different solvers!

Multiport S-parameters formal definition

$$\begin{cases} \overline{V} = (V_1, V_2, ..., V_N)^t \\ \overline{I} = (I_1, I_2, ..., I_N)^t \end{cases}$$

- vector of port voltages
- vector of port currents



$$Z_0 = diag\{Z_{0i}, i = 1,...,N\} \in C^{N \times N}$$ normalization impedances

$$\overline{a} = \frac{1}{2} Z_0^{-1/2} \cdot \left(\overline{V} + Z_0 \cdot \overline{I} \right)$$ - vector of incident waves

$$\overline{b} = \frac{1}{2} Z_0^{-1/2} \cdot \left(\overline{V} - Z_0 \cdot \overline{I} \right)$$ - vector of reflected waves

Scattering matrix definition:

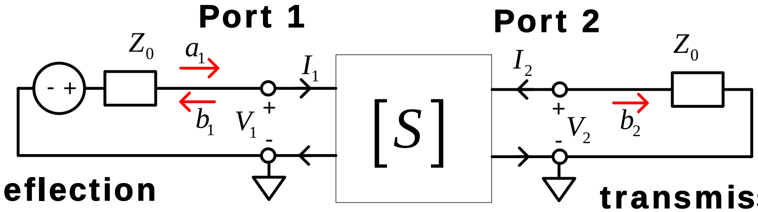
$$\overline{b} = S \cdot \overline{a}, \qquad S \in C^{N \times N}, \qquad S_{i,j} = \left. \frac{b_i}{a_j} \right|_{a_k = 0\ k \neq j}$$

Frequency Domain (FD)

Reflected wave at port i with unit incident wave at port j defines scattering parameter S[i,j]

Example of S-parameters definition

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{1,1} & S_{1,2} \\ S_{2,1} & S_{2,2} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$



reflection parameter $S_{1,1} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$

transmission parameter $S_{2,1} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$

$v_i^+ = \sqrt{Z_0} \cdot a_i$ **voltage of incident wave**

$v_i^- = \sqrt{Z_0} \cdot b_i$ **voltage of reflected wave**

$V_i = v_i^+ + v_i^-$ **total voltage**

$I_i = \frac{1}{Z_0}\left(v_i^+ - v_i^-\right)$ **total current**

$P_i^+ = |a_i|^2$ **power of incident wave**

$P_i^- = |b_i|^2$ **power of reflected wave**

$$|S_{1,1}|^2 = \frac{|b_1|^2}{|a_1|^2} = \frac{P_1^-}{P_1^+} \qquad |S_{2,1}|^2 = \frac{|b_2|^2}{|a_1|^2} = \frac{P_2^-}{P_1^+}$$

Magnitude is limited by 1 for passive systems!

$|S_{i,j}| = \sqrt{\mathrm{Re}\left(S_{i,j}\right)^2 + \mathrm{Im}\left(S_{i,j}\right)^2}$ **magnitude**

$|S_{i,j}|_{dB} = 20 \cdot \log\left(|S_{i,j}|\right)$ **magnitude in dB**

$\angle S_{i,j} = \arctan\left(\mathrm{Im}\left(S_{i,j}\right) / \mathrm{Re}\left(S_{i,j}\right)\right)$ **phase**

$i = 1, 2; \quad j = 1, 2;$

S-parameters are available in 2 forms

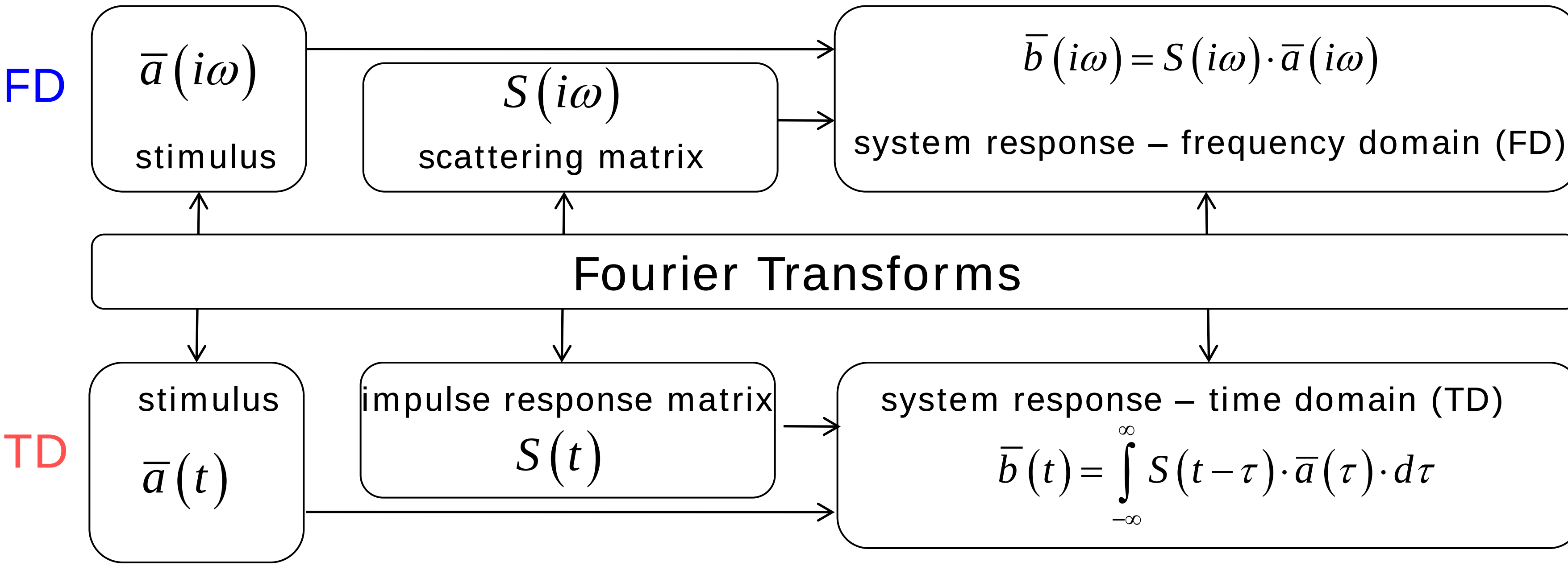
- Analytical models
  - Circuit with lumped elements (rational models)
  - Distributed circuits (models with delays)
- Tabulated (discrete) Touchstone models
  - SPICE simulators
  - Microwave analysis software
  - Electromagnetic analysis software
  - Measurements (VNA or TDNA)

Common S-parameter model defects

- Model bandwidth deficiency
  - S-parameter models are band-limited due to limited capabilities of solvers and measurement equipment
  - Model should include DC point or allow extrapolation, and high frequencies defined by the signal spectrum
- Model discreteness
  - Touchstone models are matrix elements at a set of frequencies
  - Interpolation or approximation of tabulated matrix elements may be necessary both for time and frequency domain analyses
- Model distortions due to
  - Measurement or simulation artifacts
  - Passivity violations and local “enforcements”
  - Causality violations and “enforcements”
- Human mistakes of model developers and users
- How to rate quality of the models?

System response computation requires frequency-continuous S-parameters from DC to infinity

$$S(i\omega) = \int_{-\infty}^{\infty} S(t)\cdot e^{-i\omega t}\cdot dt, \quad S(i\omega) \in C^{N\times N}$$



$$S(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(i\omega)\cdot e^{i\omega t}\cdot d\omega, \quad S(t) \in R^{N\times N}$$

Possible approximations for discrete models

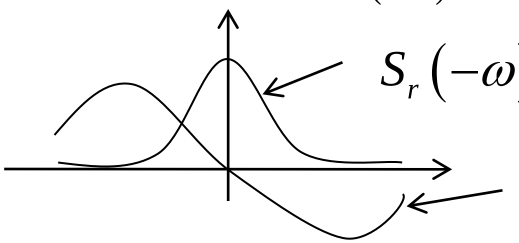
- Discrete Fourier Transform (DFT) and convolution
  - Slow and may require interpolation and extrapolation of tabulated S-parameters (uncontrollable error)
- Approximate discrete S-parameters with rational functions (RMS error)
  - Accuracy is under control over the defined frequency band
  - Frequency-continuous causal functions defined from DC to infinity with analytical impulse response
  - Fast recursive convolution algorithm to compute TD response
  - Results consistent in time and frequency domains
- Not all Touchstone models are suitable for either approach

Realness constrain on time-domain response

- Time-domain impulse response matrix must be real function of time

$$S(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(i\omega) \cdot e^{i\omega t} \cdot d\omega, \quad S(t) \in R^{N \times N}$$

- It is true if $S(i\omega) = S_r(\omega) + i \cdot S_i(\omega)$ and



$S_r(-\omega) = S_r(\omega)$ real part is even function of frequency

$S_i(-\omega) = -S_i(\omega)$ imaginary part is odd function of frequency

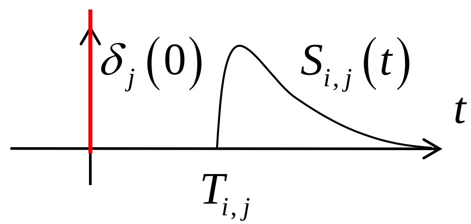
- Those conditions are satisfied by default because of we do not use negative frequencies in Touchstone models
- Conditions at zero frequency are useful to restore the DC point:

$$\left. \frac{dS_r(\omega)}{d\omega} \right|_{\omega=0} = 0, \quad S_i(0) = 0$$

DC condition for all multiport parameters

Causality of LTI system (TD & FD)

- The system is causal if and only if all elements of the time-domain impulse response matrix are $S_{i,j}(t) = 0 \ at \ t < 0$
  delayed causality (for interconnects):
  $$S_{i,j}(t) = 0 \ at \ t < T_{i,j}, \ T_{i,j} > 0$$



- This lead to Kramers-Kronig relations in frequency-domain

$$S(i\omega) = \frac{1}{i\pi} PV \int_{-\infty}^{\infty} \frac{S(i\omega')}{\omega - \omega'} \cdot d\omega', \quad PV = \lim_{\varepsilon \to 0} \left(\int_{-\infty}^{\omega-\varepsilon} + \int_{\omega+\varepsilon}^{+\infty} \right)$$

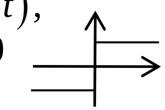
$$S_r(\omega) = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{S_i(\omega')}{\omega - \omega'} \cdot d\omega', \quad S_i(\omega) = \frac{-1}{\pi} PV \int_{-\infty}^{\infty} \frac{S_r(\omega')}{\omega - \omega'} \cdot d\omega'$$

Kramers, H.A., Nature, v 117, 1926 p. 775..
Kronig, R. de L., J. Opt. Soc. Am. N12, 1926, p 547.

derivation

$$S(t) = sign(t) \cdot S(t),$$

$$sign(t) = \begin{vmatrix} -1, & t < 0 \\ 1, & t > 0 \end{vmatrix}$$



$$S(i\omega) = F\{S(t)\} =$$

$$= \frac{1}{2\pi} F\{sign(t)\} * F\{S(t)\}$$

$$F\{sign(t)\} = \frac{2}{i\omega}$$

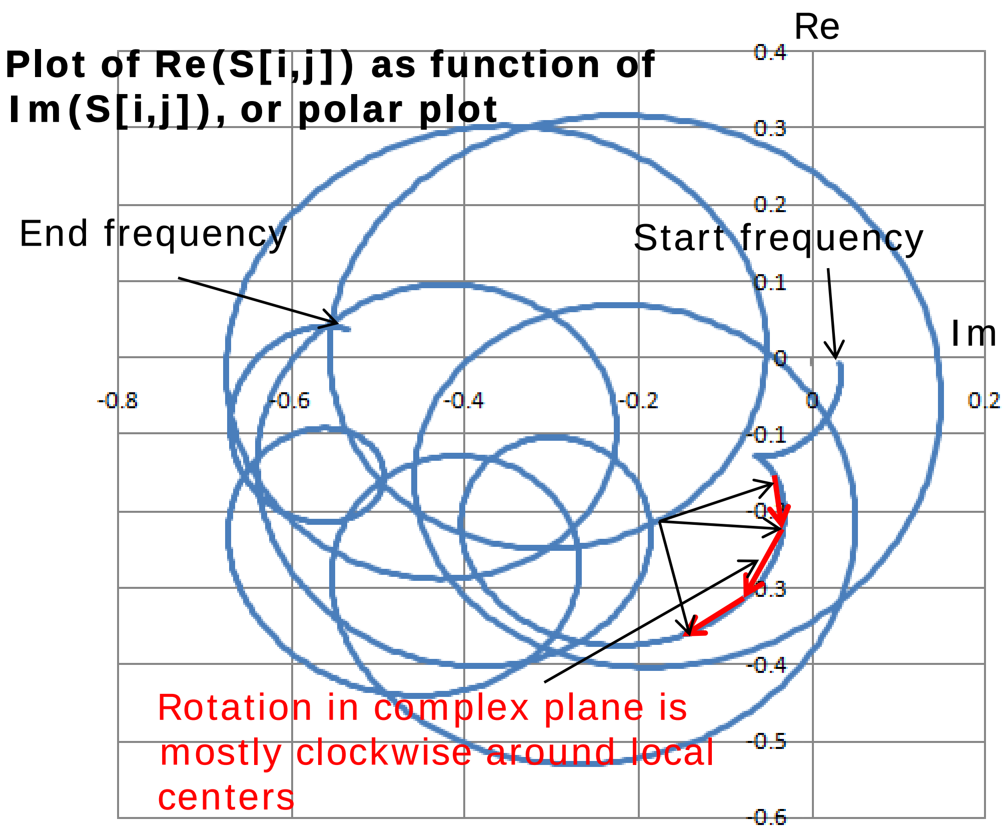
Causality estimation - difficult way

- Kramers-Kronig relations cannot be directly used to verify causality for the frequency-domain response known over the limited bandwidth at some points
- Causality boundaries can be introduced to estimate causality of the tabulated and band-limited data sets
  - Milton, G.W., Eyre, D.J. and Mantese, J.V, *Finite Frequency Range Kramers Kronig Relations: Bounds on the Dispersion,* Phys. Rev. Lett. 79, 1997, p. 3062-3064
  - Triverio, P. Grivet-Talocia S., *Robust Causality Characterization via Generalized Dispersion Relations*, IEEE Trans. on Adv. Packaging, N 3, 2008, p. 579-593.

Even if test passes – a lot of uncertainties due to band limitedness

Causality estimation - easy way

- "Heuristic" causality measure based on the observation that polar plot of a causal system rotates mostly clockwise (suggested by V. Dmitriev-Zdorov)



Causality measure (CM) can be computed as the ratio of clockwise rotation measure to total rotation measure in %.

If this value is below 80%, the parameters are reported as suspect for possible violation of causality.

Algorithm is good for numerical models (to find under-sampling), but no so good for measured data due to noise!

Stability and passivity in time-domain

- The system is stable if output is bounded for all bounded inputs

$$|a(t)| < K \Rightarrow |b(t)| < M, \ \forall t \qquad \text{(BIBO)}$$

- A multiport network is passive if energy absorbed by multiport

$$E(t) = \int_{-\infty}^{t} \left[\overline{a}^t(\tau) \cdot \overline{a}(\tau) - \overline{b}^t(\tau) \cdot \overline{b}(\tau) \right] \cdot d\tau \geq 0, \ \forall t \qquad \text{(does not generate energy)}$$

  for all possible incident and reflected waves

- If the system is passive according to the above definition, it is also causal

$$\overline{a}(t) = 0, \ \forall t < t_0 \Rightarrow \int_{-\infty}^{t} \left[\overline{b}^t(\tau) \cdot \overline{b}(\tau) \right] \cdot d\tau \leq 0 \Rightarrow \overline{b}(t) = 0, \ \forall t < t_0$$

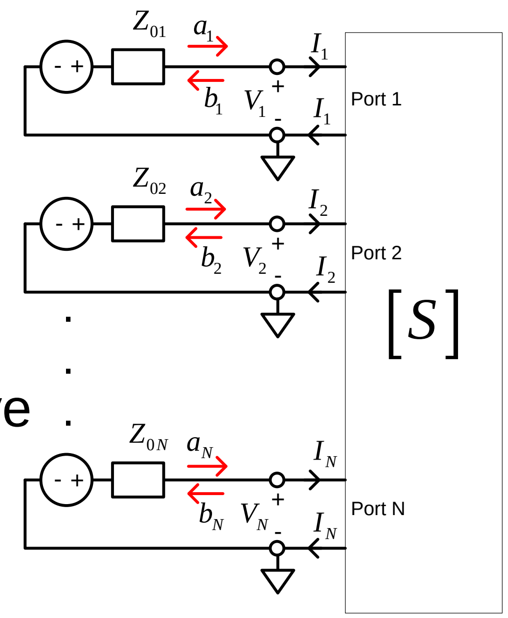
- Thus, we need to check only the passivity of interconnect system!

P. Triverio S. Grivet-Talocia, M.S. Nakhla, F.G. Canavero, R. Achar, Stability, Causality, and Passivity in Electrical Interconnect Models, IEEE Trans. on Advanced Packaging, vol. 30. 2007, N4, p. 795-808.

Passivity in frequency domain

- Power transmitted to multiport is a difference of power transmitted by incident and reflected waves: $P_{in} = \sum_{n=1}^{N} |a_n|^2 - |b_n|^2 = \left[\overline{a}^* \cdot \overline{a} - \overline{b}^* \cdot \overline{b}\right]$

  or $P_{in} = \overline{a}^* \cdot \overline{a} - \overline{a}^* \cdot S^* S \cdot \overline{a} = \overline{a}^* \cdot \left[U - S^* S\right] \cdot \overline{a}$



- Transmitted power is defined by Hermitian quadratic form and must be not negative for passive multiport for any combination of incident waves
- Quadratic form is non-negative if eigenvalues of the matrix are non-negative (Golub & Van Loan):

$$eigenvals\left[U - S^* \cdot S\right] \geq 0 \Rightarrow eigenvals\left[S^* \cdot S\right] \leq 1$$

(*U* is unit matrix)

Sufficient condition only if verified from DC to infinity (impossible for discrete Touchstone models)

Good Touchstone models of interconnects

- Must have sufficient bandwidth matching signal spectrum
- Must be appropriately sampled to resolve all resonances
- Must be reciprocal (linear reciprocal materials used in PCBs)

$$S_{i,j} = S_{j,i} \quad or \quad S = S^t$$

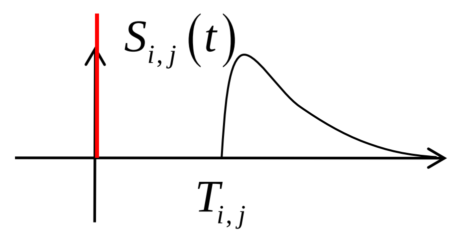
- Must be passive (do not generate energy)

$$P_{in} = \overline{a}^* \cdot \left[U - S^* S \right] \cdot \overline{a} \geq 0 \Rightarrow eigenvals\left[S^* \cdot S \right] \leq 1$$

from DC to infinity!

- Have causal step or impulse response (response only after the excitation)

$$S_{i,j}(t) = 0, \quad t < T_{ij}$$



Quality metrics (0-100%) to define goodness

First introduced at IBIS forum at DesignCon 2010

- Passivity Quality Measure:

$$PQM = \max\left[\frac{100}{N_{total}}\left(N_{total} - \sum_{n=1}^{N_{total}} PW_n\right), 0\right]\%$$

$$PW_n = 0 \ if \ \ PM_n < 1.00001; \ otherwise \ PW_n = \frac{PM_n - 1.00001}{0.1}$$

$$PM_n = \sqrt{\max\left[eigenvals\left(S^*\left(f_n\right)\cdot S\left(f_n\right)\right)\right]}$$

should be >99%

- Reciprocity Quality Measure:

$$RQM = \max\left[\frac{100}{N_{total}}\left(N_{total} - \sum_{n=1}^{N_{total}} RW_n\right), 0\right]\%$$

$$RW_n = 0 \ if \ \ RM_n < 10^{-6}; \ otherwise \ RW_n = \frac{RM_n - 10^{-6}}{0.1}$$

$$RM_n = \frac{1}{N_s}\sum_{i,j}\left|S_{i,j}\left(f_n\right) - S_{j,i}\left(f_n\right)\right|$$

should be >99%

- Causality Quality Measure: Minimal ratio of clockwise rotation measure to total rotation measure in % (should be >80% for numerical models)

Preliminary quality estimation metrics

- Preliminary Touchstone model quality can be estimated with Passivity, Reciprocity and Causality quality metrics (PQM, RQM, CQM)

Metric/Model Icon	- good	- acceptable	- inconclusive	- bad
Passivity	[100, 99.9]	(99.9, 99]	(99, 80]	(80, 0]
Reciprocity	[100, 99.9]	(99.9, 99]	(99, 80]	(80, 0]
Causality	[100, 80]	(80, 50]	(50, 0]	-----

Color code	Passivity (PQM)	Reciprocity (RQM)	Causality (CQM)
Green – good	[99.9, 100]	[99.9, 100]	[80, 100]
Blue – acceptable	[99, 99.9)	[99, 99.9)	[50, 80)
Yellow – inconclusive	[80, 99)	[80, 99)	[20, 50)
Red - bad	[0, 80)	[0, 80)	[0, 20)

Example of preliminary quality estimation in Simbeor Touchstone Analyzer™

Small passivity & reciprocity violations in most of the models
Low causality in some measured data due to noise at high frequencies

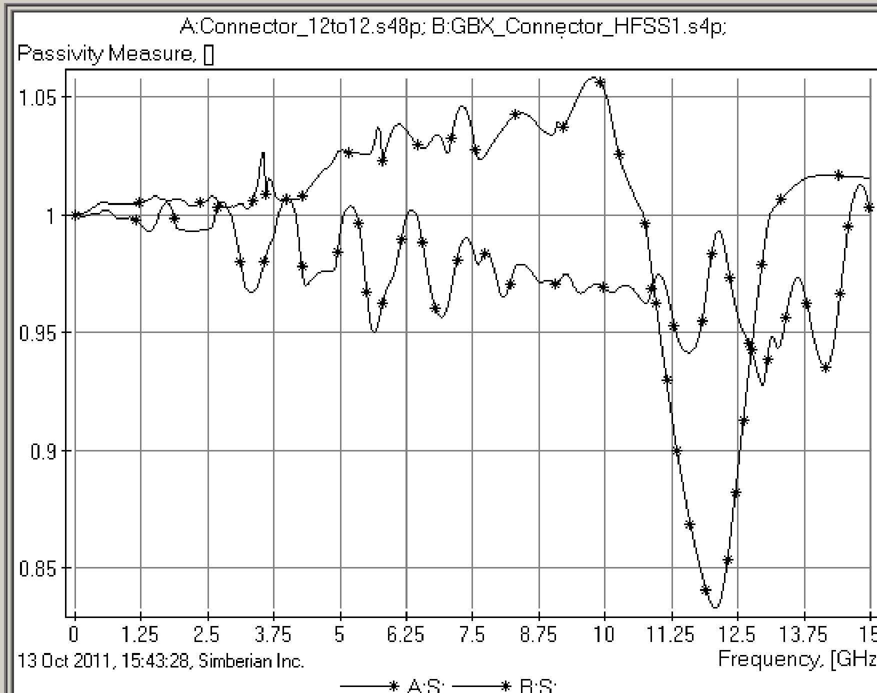
Simbeor - [Touchstone Analyzer]

Touchstone Analyzer

Refresh | Tree-view | Table-view

File name	Quality	Passivity	Reciprocity	Causality
3coupleddiffpairJ74J76toJ7...	-	95.9	99.3	0
3coupleddiffpairJ74J76toJ8...	-	96	99.2	0
beattystandard25ohm.S2P	-	100	95.5	12.7
beattystandard80ohm.S2P	-	100	95.6	0
Connector_12to12.s48p	-	82.6	100	80.5
crosstalkstriptobroadsidec...	-	96.4	99.2	0
crosstalkstriptobroadsidec...	-	96.3	99.2	0
crosstalkstriptobroadsidec...	-	96.1	99.3	0
crosstalkstriptobroadsidec...	-	96.1	99	0
crsstalkstriplinedpair.S4P	-	97.3	99.2	35.2
example calibration.S2P	-	99.2	99.5	0
GBX_Connector_HFSS1.s4p	-	99.5	91	96.2
IConect.s2p	-	99.4	52.6	73.4
J104J94-J105J95.s4p	-	99.8	100	97.7
J104J94-J97J99.s4p	-	100	100	98
J20J24-J21J25-6in.s4p	-	99.4	100	94.3
J20J24-J21J25.s4p	-	99.4	100	94.3
J26J30-J27J31-3in.s4p	-	99.5	99.9	92.7
J26J30-J27J31.s4p	-	99.5	99.9	92.7

Ready



Rational approximation of S-parameters as the frequency-continuous model

$$\bar{b} = S \cdot \bar{a},\quad S_{i,j} = \left.\frac{b_i}{a_j}\right|_{a_k=0\ k\neq j} \Rightarrow S_{i,j}(i\omega) = \left[d_{ij} + \sum_{n=1}^{N_{ij}} \left(\frac{r_{ij,n}}{i\omega - p_{ij,n}} + \frac{r^*_{ij,n}}{i\omega - p^*_{ij,n}} \right) \right] \cdot e^{-s \cdot T_{ij}}$$

Continuous functions of frequency defined from DC to infinity

$s = i\omega,\ d_{ij} - values\ at\ \infty,\ N_{ij} - number\ of\ poles,$

$r_{ij,n} - residues,\ p_{ij,n} - poles\ (real\ or\ complex), T_{ij} - optional\ delay$

- Pulse response is analytical, real and delay-causal:

$$S_{i,j}(t) = 0,\ t < T_{ij}$$

$$S_{i,j}(t) = d_{ij}\delta(t - T_{ij}) + \sum_{n=1}^{N_{ij}} \left[r_{ij,n} \cdot \exp\left(p_{ij,n} \cdot (t - T_{ij}) \right) + r^*_{ij,n} \cdot \exp\left(p^*_{ij,n} \cdot (t - T_{ij}) \right) \right],\ t \geq T_{ij}$$

- Stable $\mathrm{Re}\left(p_{ij,n}\right) < 0$
- Passive if $eigenvals\left[S(\omega) \cdot S^*(\omega) \right] \leq 1\ \forall \omega,\ from\ 0\ to\ \infty$
- Reciprocal if $S_{i,j}(\omega) = S_{j,i}(\omega)$

May require enforcement

Bandwidth and sampling for rational approximation

- If no DC point, the lowest frequency in the sweep should be
  - Below the transition to skin-effect (1-50 MHz for PCB applications)
  - Below the first possible resonance in the system (important for cables, L is physical length)

$$L < \frac{\lambda}{4} = \frac{c}{4 f_l \cdot \sqrt{\varepsilon_{eff}}} \Rightarrow f_l < \frac{c}{4L \cdot \sqrt{\varepsilon_{eff}}}$$

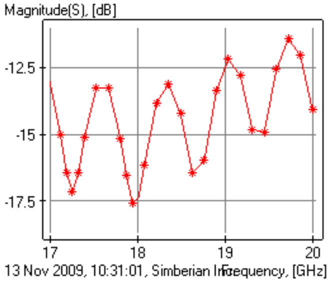
- The highest frequency in the sweep must be defined by the required resolution in time-domain or by spectrum of the signal (by rise time or data rate)

$$f_h > \frac{1}{2t_r}$$

- The sampling is very important for DFT and convolution-based algorithms, but not so for algorithms based on fitting

$$df < \frac{c}{4L \cdot \sqrt{\varepsilon_{eff}}}$$

  - There must be 4-5 frequency point per each resonance
  - The electrical length of a system should not change more than quarter of wave-length between two consecutive points



Rational approximation can be used to

- Compute time-domain response of a channel with a fast recursive convolution algorithm (exact solution for PWL signals)
- Improve quality of tabulated Touchstone models
  - Fix minor passivity and causality violations
  - Interpolate and extrapolate with guarantied passivity
- Produce broad-band SPICE macro-models
  - Smaller model size, stable analysis
  - Consistent frequency and time domain analyses in any solver
- Measure the original model quality

Final quality estimation

- Accuracy of discrete S-parameters approximation with frequency-continuous macro-model, passive from DC to infinity

$$RMSE = \max_{i,j}\left[\sqrt{\frac{1}{N}\sum_{n=1}^{N}\left|S_{ij}(n) - S_{ij}(\omega_n)\right|^2}\right]$$

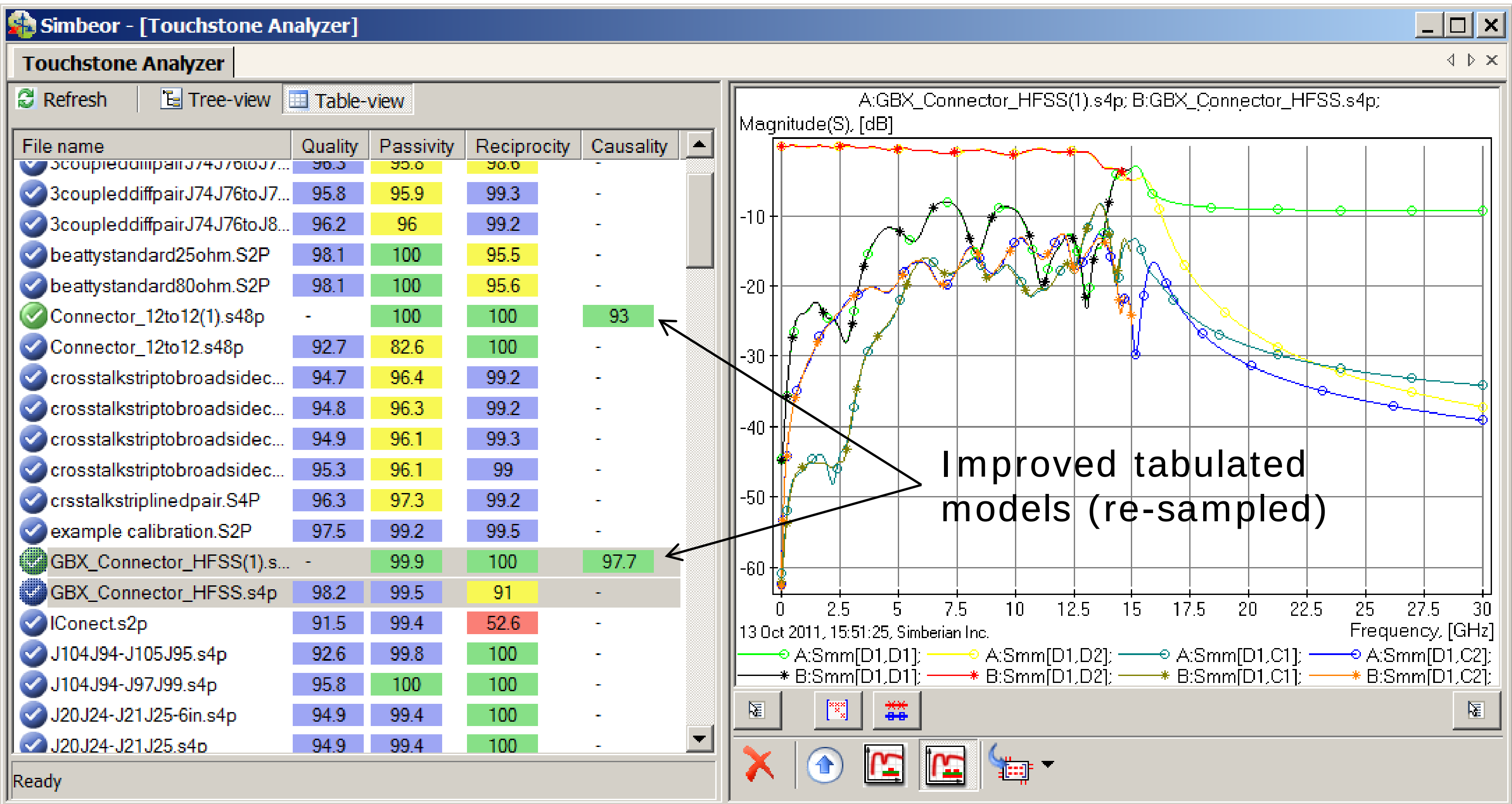
- Can be used to estimate quality of the original data

$$Q = 100 \cdot \max\left(1 - RMSE, 0\right)\%$$

Model Icon/Quality	Quality Metric	RMSE
- good	[99, 100]	[0, 0.01]
- acceptable	[90, 99)	(0.01, 0.1]
- inconclusive	[50, 90)	(0.1, 0.5]
- bad	[0, 50)	> 0.5
- uncertain	[0,100], not passive or not reciprocal	

Example of final quality estimation in Simbeor Touchstone Analyzer®

All rational macro-models are passive, reciprocal, causal and have acceptable accuracy (acceptable quality of original models)



File name	Quality	Passivity	Reciprocity	Causality
3coupleddiffpairJ74J76toJ7...	96.3	95.8	98.6	-
3coupleddiffpairJ74J76toJ7...	95.8	95.9	99.3	-
3coupleddiffpairJ74J76toJ8...	96.2	96	99.2	-
beattystandard25ohm.S2P	98.1	100	95.5	-
beattystandard80ohm.S2P	98.1	100	95.6	-
Connector_12to12(1).s48p	-	100	100	93
Connector_12to12.s48p	92.7	82.6	100	-
crosstalkstriptobroadsidec...	94.7	96.4	99.2	-
crosstalkstriptobroadsidec...	94.8	96.3	99.2	-
crosstalkstriptobroadsidec...	94.9	96.1	99.3	-
crosstalkstriptobroadsidec...	95.3	96.1	99	-
crsstalkstriplinedpair.S4P	96.3	97.3	99.2	-
example calibration.S2P	97.5	99.2	99.5	-
GBX_Connector_HFSS(1).s...	-	99.9	100	97.7
GBX_Connector_HFSS.s4p	98.2	99.5	91	-
IConect.s2p	91.5	99.4	52.6	-
J104J94-J105J95.s4p	92.6	99.8	100	-
J104J94-J97J99.s4p	95.8	100	100	-
J20J24-J21J25-6in.s4p	94.9	99.4	100	-
J20J24-J21J25.s4p	94.9	99.4	100	-

Conclusion: How to avoid problems with S-parameter models?

- Use reciprocity, passivity and causality metrics for preliminary analysis
  - RQM and PQM metrics should be > 99% (acceptable level)
  - CQM should be > 80% for all causal numerical models
- Use the rational model accuracy as the final quality measure
  - QM should be > 90% (acceptable level)
- Discard the model with low RQM, PQM and QM metrics!
  - The main reason is we do not know what it should be
- Models that pass the quality metrics may still be not usable or mishandled by a system simulator
  - Due to band-limitedness, discreteness and brut force model fixing
- Use rational or BB SPICE macro-models instead of Touchstone models for consistent time and frequency domain analyses

Contact and resources

- Yuriy Shlepnev, Simberian Inc.

  shlepnev@simberian.com

  Tel: 206-409-2368

- To learn more on S-parameters quality see the following presentations (also available on request):
  - Y. Shlepnev, Quality Metrics for S-parameter Models, DesignCon 2010 IBIS Summit, Santa Clara, February 4, 2010
  - H. Barnes, Y. Shlepnev, J. Nadolny, T. Dagostino, S. McMorrow, Quality of High Frequency Measurements: Practical Examples, Theoretical Foundations, and Successful Techniques that Work Past the 40GHz Realm, DesignCon 2010, Santa Clara, February 1, 2010.
  - E. Bogatin, B. Kirk, M. Jenkins, Y. Shlepnev, M. Steinberger, How to Avoid Butchering S-Parameters, DesignCon 2011
  - Y. Shlepnev, Reflections on S-parameter quality, DesignCon 2011 IBIS Summit, Santa Clara, February 3, 2011

