Modeling frequency-dependent conductor losses and dispersion in serial data channel interconnects

Yuriy Shlepnev
Simberian Inc., www.simberian.com

Abstract: Models of transmission lines and transitions accurate over 5-6 frequency decades are required to simulate interconnects for multi-gigabit serial data channels. Extremely broadband modeling of conductor properties for such high-speed channels is a particularly challenging task for electromagnetic tools. This paper discusses the conductor effects in general and the approximations used in different simulation technologies and tools.

Introduction

Faster data rates drive the need for accurate electromagnetic models for multi-gigabit serial data channels. Both early exploration of the interconnect budget and channel verification require frequency-dependent ultra-broadband models for transmission lines and via holes extracted with a 3D full-wave electromagnetic simulator. Without these advanced models, a channel design may require experimental verification and iterations to improve overall performance, adding delays and increased costs to the project.

Static and magneto-static 2D field solvers are typically used to design high-speed nets, but they are not quite suitable to design multi-gigabit channel due to multiple limitations discussed in this paper. The industry is gradually becoming aware of these deficiencies and a transition from static to 3D full-wave tools takes place. Unfortunately, 3D and full-wave analysis does not automatically guarantees that all conductor-related effects are accounted for. Solvers originally developed for microwave applications for instance are optimized for analysis of relatively narrow-band systems and for frequencies where skin-effect is well developed. Such models are not valid at low and intermediate frequencies where there is no skin-effect or skin-effect is not developed yet and the models have to include those frequencies for digital applications. As a result, a broadband models of interconnects may show non-physical behavior of extracted inductance and resistance at low and intermediate frequencies or violate causality requirements. In other words, the transition to skin effect in traces and metal planes is usually neglected in 3D tools in general. The frequency-dependent skin-effect is usually roughly approximated or not accounted for at all in 3D full-wave time-domain tools and in some PEEC-based tools. Another differentiator of PCB and packaging interconnects is large conductor surface roughness that is neglected by almost all electromagnetic and static tools. Many 3D tools do not have built-in capabilities to extract modal and per unit length parameters of transmission lines. This paper discusses the limitations of different simulation techniques for conductor effects and requirements for broad-band transmission line parameters extraction tools.
Absorption of energy by conductors and skin-effect

Multiconductor transmission line models are usually used to simulate interconnects in packages and on PCBs. Quasi-TEM waves propagating in these lines have electric and magnetic field components predominantly in the line cross-section and the signal is transmitted along the line by these fields. In addition, the fields may have components along the line if there is inhomogeneous or multilayered dielectric and also in the case of non-ideal conductors discussed here. Non-ideal conductors absorb the electromagnetic energy and cause unwanted loss and dispersion that have to be accounted for in the transmission line analysis. Mathematically, fields inside the conductors can be described with Maxwell’s equations without the displacement term as a magneto-quasi-static or diffusion problem with very high accuracy. Locally, on the surface of the conductor, electromagnetic field can be approximated as plane waves transmitting the energy inside the conductor that produces non-uniform conductor current distribution as illustrated for a simple case in Fig. 1. Parameter \( \delta \) of the complex exponential function is called skin depth and can be calculated as

\[
\delta = \frac{\rho}{\pi \mu f} \text{[m]}
\]

where \( \rho \) is conductor bulk resistivity in [Ohm-m], \( \mu \) is permeability in [H/m] and \( f \) is frequency in [Hz]. \( \rho \) for annealed copper is about \( 1.724 \times 10^{-8} \text{[Ohm-m]} \) for instance. The skin depth \( \delta \) becomes smaller at higher frequencies and eventually becomes much smaller than the conductor cross-section and we can call this state as well-developed skin-effect. Note that fields and currents in conductors with complex shapes can be approximated as superposition of such plane waves from DC to any given frequency.

\[
J_y = J_z \cdot \exp \left( \frac{- (1 + i) x}{\delta_s} \right)
\]

\[
E_y = \rho J_y
\]

Fig. 1. Local current density and electric field distribution near a conductor surface parallel to the ZY-plane. Transmission line is oriented along the Y-axis, and conductor cross-section is in ZX-plane. Current is decreasing as a complex exponential function. Electric field component \( E_y \) along the conductor is proportional to the current flowing along the conductor according to Ohm’s law. The Poynting’s vector points in the direction of the conductor interior.

Qualitative view of conductor effects on signal propagation

Considering multiconductor lines, there are three major factors defining distribution of currents in the conductors and corresponding loss and dispersion. Those are the conductor bulk resistivity, shape of the conductors and proximity of the conductors (strip and plane for instance). In addition surface roughness and dielectrics surrounding the conductors alter the conductive properties of the traces. The frequency-dependent conductor effects can be qualitatively separated into three frequency regions – low, medium and high [1], [2]. This
classification can be based on the state of current density distribution in conductors [1] or on the base of properties of the conductor impedance per unit length (p.u.l.) [2]. Note that any classification is necessary only to understand the limitations of the models that include only some effects and do not take into account the others. Uniform current distribution in homogeneous trace and plane conductors take place from DC to some relatively low frequency (it is not valid for composite conductors). The corresponding impedance p.u.l. has almost constant real part $R_{DC}$ and inductive part with almost constant inductance $L_{DC}$ that characterizes the uniform distribution of magnetic field inside both strips and planes as illustrated in Fig. 2.

![Fig. 2 Current density changes with growth of frequency. At DC frequencies current is uniformly distributed over the strip and plane conductors. At relatively low frequency currents concentrates below the strip due to proximity effect. At the medium frequencies, current density becomes larger near surface of the conductors and at the strip edges. Finally, at high frequencies the skin effect is well-developed and roughness and dispersion plus edge effect may further contribute to the resistance growth and change the inductance p.u.l.](image)

Current distribution in conductors may start changing at frequencies as low as 10 KHz where plane current concentrates below the strip to minimize the energy absorption by the plane (though these losses grow with the frequency). At medium frequency range, current density re-distribution takes place. Corresponding effects are not separable but can be called by different names like proximity or crowding effect, edge effect, transition to skin-effect as also illustrated in Fig. 2. At medium frequencies the resistance p.u.l. increases and inductance p.u.l. decreases because of shrinking conductor area with the current and magnetic field. Skin and edge effects become visible at higher frequencies and, finally, at relatively high frequencies, most of the current flow in a thin surface layer and corresponding state can be characterized as the well-developed skin effect. Without edges and dielectrics the resistance at high frequencies grows as $\sqrt{f}$ and conductor internal inductance decreases as $1/\sqrt{f}$.

Interaction of non-homogeneous dielectric dispersion and edge or proximity effects further accelerate the growth of resistance p.u.l. with frequency [2]. This is valid even for ideal
dielectrics. In addition, roughness may considerably increase the resistance p.u.l. at high frequencies [3].

**Transition to skin-effect**

Though, conductor shape and dielectric properties may have significant effect on the energy absorption in the conductors, we will try to determine the medium or transition frequencies for different technologies that use strip or microstrip lines. This can be done on the base of strip or plane thickness ratio to the skin depth. If conductor thickness is less or equal to $0.5 \cdot \delta$ (half skin-depth) the skin effect is not visible as illustrated in Fig. 3 for a typical PCB trace. At a frequency when the strip thickness becomes equal to $2 \cdot \delta$, the skin and edge effect become visible as shown in Fig. 3, but it is not well developed skin effect yet. According to rule of thumb in microwave engineering, the skin effect becomes well developed only if conductor thickness becomes equal to $5 \cdot \delta$ (five skin-depth).

Alternatively, the high transition frequency can be derived on the base of comparison of the real and imaginary parts of internal conductor impedance p.u.l. [2]. The low transition frequency may be defined by deviation of imaginary part from zero. The transition ends as soon as the real and imaginary parts are equal with some tolerance. For the strip example shown in Fig. 3., the impedances p.u.l. are $Z(1.7\text{MHz}) = 2.67 + i0.11$, $Z(28\text{MHz}) = 2.95 + i1.71$, and $Z(170\text{MHz}) = 6.44 + i6.22 \text{ [Ohm/m]}$. It is clear that at 28 MHz, where strip is two skin depths, the imaginary part is almost 40% different from the real part and it is still the medium frequency. **Note that the conductor impedance real and imaginary parts may deviate from each other at higher frequencies due to the dispersion and edge effects.**

We can generalize this observation on the transition as shown in Fig. 4. Strip or plane thicknesses on PCBs and in packages can range from 50 um to 1 um and corresponding transition frequencies range from 1 MHz to above 10 GHz.
Conductor surface roughness

Roughness is what makes serial data interconnects modeling considerably different from the analysis of microwave integrated circuits for instance. Polishing of conductor and dielectric surfaces is not a possibility for printed-circuit boards, and roughness can increase the interconnect loss as much as 50% as shown in [3]. There are multiple methods developed to model the roughness and Fig. 5 provides guidelines to determine at what frequency the roughness has to be taken into account in the interconnect analysis. As soon as the roughness root mean square (rms) value becomes comparable with the skin depth, it must be accounted. Some manufacturing technologies give 10 um rms roughness that degrades the signal at frequencies as low as 40 MHz. In contrast, modeling interconnects with 0.5 um rms roughness may not require the roughness models at frequencies below 18 GHz.
Fig. 5. Ratio of rms surface roughness values to skin depth in copper. Brown line – 10 um, magenta – 5 um, black 1 um, blue 0.5 um, cyan – 0.1 um. Roughness must be taken into account in model if the ratio is about 1 or greater. PCB and packaging interconnects may have rms roughness values between 10 um and 0.5 um.

**Static and quasi-static field solvers**

A static field simulator solves Laplace’s equations for electrostatic potential in 2D cross-sections of multiconductor transmission lines. Boundary element method with meshing of conductor surfaces and dielectric boundaries or finite element methods are usually used in commercial static field solvers. The result of a static solver analysis is always a capacitance matrix per unit length and charge distribution on the conductor surfaces. **All other parameters are derived from these.** To find an external inductance matrix per unit length \( L_{ext} \) for instance, capacitance matrix \( C(\varepsilon_0) \) has to be calculated for the problem without dielectrics and then the inductance is calculated as \( L_{ext} = \mu_0 \varepsilon_0 C(\varepsilon_0)^{-1} \). Here \( \varepsilon_0 \) and \( \mu_0 \) are permittivity and permeability of the vacuum. Conductors are assumed to be ideal (no resistivity) in all computations. To estimate conductor losses and conductor internal inductance, charge distribution on the conductor surface is calculated for the original problem with dielectric layers. Then, assuming well-developed skin-effect in all conductors, perturbation technique is used to calculate resistance per unit length \( R_s(f_0) \) at a given frequency \( f_0 \) (it is usually 1 GHz in commercial field solvers). In addition \( R_{DC} \) is calculated as diagonal matrix with resistances per unit length of all strips on the diagonal. Finally, Wheeler’s assumption about the equality of the real and imaginary parts of conductor internal impedance p.u.l. is used to approximation the total frequency-dependent impedance p.u.l. as follows:

\[
Z_w(f) = R_{DC} + (1+i)R_s(f_0) \sqrt{\frac{f}{f_0} + i2\pi f \cdot L_{ext}} \left[ \frac{Ohm}{m} \right]
\]
$R_{DC}$, normalized skin-effect resistance $R_{sn}$ = \( \frac{R_s(f_0)}{\sqrt{f_0}} \left[ \frac{Ohm}{m/\text{Hz}} \right] \) and $L_{ext}$ (or inductance at infinity) are all components of the conductor-related loss and dispersion models in W-element or similar transmission line models in system-level solvers. There are multiple problems with this approach. First of all, $R_{DC}$ usually does not include the reference plane resistance, and in addition it creates inequality of the real and imaginary parts of the internal impedance at high frequency, that contradicts to the Wheeler’s rule. Second of all, it does not capture behavior of the impedance at transitional frequencies when resistance and inductance p.u.l. just begin to deviate from the DC values. In addition, the internal inductance goes to infinity at DC as $1/\sqrt{f}$ instead of converging to the inductance at DC that is basically unknown in this approach. High-frequency approximation in problems with dielectrics is also not valid because of the current redistribution due to the dispersion and edge effects that are not accounted in the static solution (it requires full-wave solution). It basically explains why microwave engineers for instance do not use such approximation and the static field solvers in general.

To extract frequency-dependent resistance $R(f)$ and inductance $L(f)$ matrices p.u.l. a 2D magneto-quasi-static field solver can be used. It solves frequency-independent Laplace’s equations outside the conductors and frequency-dependent diffusion equation inside the conductors to model the skin effect. Hybrid technique with filaments inside the conductor and boundary element method outside the conductor or finite-element method are usually used to solve the problem. Though it can model conductor current re-distribution in the transition to skin-effect frequency range, the extraction of R and L at frequencies with well-developed skin-effect may be computationally very expensive. A finite element or filament size in the conductor interior has to be a fraction of the skin depth at the highest frequency. Otherwise, the artificial resistance saturation effect takes place when the resistance approaches some value and does not grow with the frequency after that. To extract parameters of a line up to 10 GHz, the element or filament size near the metal surface has to be at least $1/4$ or skin depth that is about 0.16 um. It would be required about 236000 elements to mesh interior of 10 mil by 1 mil trace for instance and in addition interior of one or two planes have to be meshed too (another two million elements may be required). Even if those limits are alleviated with a non-uniform conductor interior meshing, the solution does not include effect of non-uniform dielectric on the current distribution or dispersion and edge effect (magneto-quasi-static problem does not include dielectrics at all). In general magneto-quasi-static solvers are suitable for extraction of $R(f)$ and $L(f)$ from DC to the transitional frequencies and only in case if there is not much dispersion from inhomogeneous dielectrics.

2D field solvers cannot extract parameters of transitions and via-holes. Both static and magneto-quasi-static solvers do not simulate effect of roughness.

3D full-wave solvers

A 3D full-wave simulator solves Maxwell’s equations for three-dimensional (3D) problems with all six components of electromagnetic field and with the displacement term. Note that most of 3D PEEC-based tools usually neglect the displacement and hence radiation and can not be classified as full-wave. Unfortunately, full-wave does not guarantee that all
**conductor-related signal degradation effects are appropriately accounted for.**

Approximation of the conductor-related frequency-dependent effects is easier to do in frequency-domain (FD). It is more difficult to do it in time-domain (TD) in general. Though, simulation in both FD and TD can be done equally accurate and there are multiple methods developed to account for all effects. In reality the results of FD and TD analyses may be considerably different due to the approximation in a particular tool. Commonly used approximations and trade offs are discussed here.

Surface impedance boundary condition (SIBC) is probably most popular model of the conductor effects and is used in most of the frequency-domain tools. It relates the electric and magnetic fields on the surface of the conductor and can be expressed as $E_s = Z_s(f)H_z$ for the conductor surface shown in Fig. 1. Here $Z_s(f) = (1+i)\frac{\rho}{\delta_s}[\text{Ohm/m}]$ is the conductor surface impedance and $\delta_s = \sqrt{\frac{\rho}{\pi\mu f}}[\text{m}]$ is the skin depth, $\rho$ is the conductor bulk resistivity.

Note that SIBC has been derived for the cases of the conductor thickness much larger than the skin depth (at least five skin depths is required) and the limitations follow from the formula for the surface impedance. The real part of $Z_s(f)$ approaches to zero proportionally to $\sqrt{f}$. It means no DC solution or all conductors become ideal at lower frequencies. Another limitation is that the inductive part of $Z_s(f)$ can be expressed as inductance that goes to infinity as $1/\sqrt{f}$ as frequency approaches to zero. **Thus, neither DC nor the transition to skin-effect can be simulated with the SIBC.** In addition, the SIBC itself may be approximated in some electromagnetic solvers based on only electric field formulation for instance. It may further degrade the conductor model quality even at high frequencies. An alternative is to model the conductor interior. In reality, this is much more expensive than in the magneto-quasi-static solvers mentioned earlier if regular finite elements are used for instance, especially taking into account that both strip and plane conductors have to be meshed. The elements inside the conductor have to be a fraction of the skin-depth at the highest frequency. Otherwise with the under-meshing of the conductor interior the resistance or attenuation saturation effect may be observed [6]. Note that the same element sizes have to be maintained over the frequency band of interest to maintain continuity and causality of the extracted parameters.

**Problems with electromagnetic simulation of conductor properties arise from huge differences in material properties between air, dielectrics and conductors.** There are two ways to address it. The first one is to use special finite elements built with the basis function constructed for a particular medium. Trefftz finite elements (see [4] for instance) use plane waves in dielectric or in conductor as the basis functions instead of linear or polynomial approximations as in the regular finite elements. It allows simulate media with incommensurable properties with high accuracy taking into account propagation and attenuation specific to a material. It also provides correct asymptotes at DC and at infinite frequencies without the effect of saturation typical for the finite element solvers. At high frequency such elements degenerate into simple SIBC. The second approach is to use hybrid simulation technology by combining method efficient for multilayered dielectrics for instance with a method efficient for the conductor interior. It can be done matching
differential impedance operator calculated by Trefftz finite elements for the strip conductor interior with an impedance operator obtained by integral equation or similar technique for the multi-layered media. Though there are no commercial solvers based solely on the Trefftz finite elements, there is one based on the hybrid simulation technology with the intra-conductor meshing [5]. Note that Trefftz finite elements were originally discovered by V.V. Nikol’skiy in late 70-s and recently re-discovered as ultra-week discontinuous Galerkin’s elements. The hybrid model with Trefftz elements inside the conductor allows adjustment of the conductor impedance visible from outside if the conductor surface is rough. Roughness model becomes frequency-dependent with this approach and can be adjusted for different roughness profiles or models. Alternatively roughness can be modeled as adjustment to conductor resistivity – it affects the conductor model at all frequencies and does not provide correct DC asymptotes. Such approximation may be used for analysis of narrow-band systems only.

Considering electromagnetic simulation in time-domain, finite difference or finite integration methods are the most commonly used in commercial solvers. To compute frequency-dependent multiport parameters of a structure, TD solvers use excitation with a Gaussian pulse and FFT to convert the response into the frequency domain. The TD solvers are usually very fast and can solve even system-level problems without channel de-composition into components. Unfortunately, it can be done only with very crude approximation of the frequency-dependent conductor effects in the time-domain. Either lossless or frequency-independent loss models are usually used to simulate a hole net or channel. Though the result of such simulation may be interesting from EMI point of view, they are usually useless for the signal integrity analysis. Even classical SI analysis with a static field solver and W-element models may provide more accurate results than a 3D electromagnetic analysis with the frequency-independent loss models (in case if no via-holes are involved). Simple frequency-dependent SIBC may become a challenging problem for a TD solver if a response with a bandwidth over 5-6 frequency decades is required for instance. $\sqrt{f}$ function is what makes the SIBC or other conductor-related boundary conditions inconvenient for the TD solvers. The function can not be directly integrated in time-domain and some approximation is required. Local multi-pole expansion and recursive convolution is the most popular and universal technique to treat SIBC and other frequency-dependent conditions (thin planes, strips, wires, dielectrics and so on). It works well for narrow band problem where just one pole is usually required that does not degrade much the simulation speed. For a broad-band analysis, at least one pole per decade has to be used that gives 5-6 poles per each cell interfacing the conductor for a system to be analyzed over 5-6 frequency decades. It may considerably slow down the simulation (1-2 orders of magnitude) or reduce the problem size to a line segment or to a single discontinuity. Note that special cells to increase accuracy of strip conductor approximation are constructed and used in some solvers. Though, it may increase the accuracy of the effective or visible strip impedance for lossless problems, to account for transition to skin-effect and skin effect in the strip the same rule on the number of poles in a local multi-pole approximation have to be applied to these cells. Roughness makes things even more complicated and is usually neglected in the TD tools.

The results of analysis of a line segment with either FD or TD electromagnetic solver are usually multiport parameters such as scattering, impedance or admittance matrices. Note that most of the solvers do not automatically extract modal and p.u.l. parameters of transmission lines from a 3D analysis. It means that if your channel contains N line
segments and all have different lengths, N different simulations have to be done and N multiport parameters used in a system level de-compositional analysis. Analysis of a channel as a whole is practically useless procedure because of long simulation time and degraded accuracy mentioned above. To my knowledge, only Simbeor solver allows extraction of modal and frequency-dependent RLGC parameters directly from 3D full-wave analysis [5]. The same RLGC parameters can be reused to analyze multiple line segments with the same cross-section.

Practical examples of 3D full-wave extraction

To illustrate the conductor effects in different frequency bands we extract transmission line parameters of a PCB microstrip line and a packaging application strip line with a hybrid 3D full-wave simulator Simbeor [5]. The solver uses method of lines for multilayered dielectrics and interior of conductive planes and Trefftz finite elements to simulate strip conductor interior.

The first example is a 7 mil wide and 1.6 mil thick microstrip line with dielectric substrate 4 mil thick and lossless dielectric with relative permittivity 4. Plane thickness is 2 mil. According to our classification the medium frequency range for such line is from 0.66 MHz to 66 MHz (strip thickness ranges from 0.5 to 5 skin depths). The results of the transmission line parameters extraction from 1 MHz to 100 GHz (5 decades) are shown in Fig. 6. Effects of transition to skin-effect, skin-effect, roughness and high-frequency dispersion can be observed on the graphs.

The second example is a 79 um wide and 5 um thick strip in lossless homogeneous dielectric with Dk=3.4. Distance to top plane 60 um, to bottom plane 138 um. Top plane thickness is 10 um and the bottom plane thickness is 15 um. The extraction results are shown in Fig. 7. This example was taken from [6], only dielectric losses are not accounted for here to
distinguish the conductor-related effects. Note that the conductor surface rms roughness in this example was different for the opposite surfaces of the strip as shown in [6]. The transition to skin effect in this example takes place at frequencies from 43 MHz to 4.3 GHz that is clearly visible on the attenuation graph for instance.

**Fig. 7.** Attenuation p.u.l. and characteristic impedance (left graph) and inductance and resistance p.u.l. (right graph) of a 79 um wide and 5 um thick strip line (packaging example). Attenuation in line with flat conductor surfaces and with rms roughness 1 um for planes and one side of the strip (the opposite side of strip was flat) are compared on the left graph. Inductances of line with lossless conductor, with flat lossy and with rough conductor are compares on the right graph.

If a static solver is used for the first PCB example and loss parameters extracted at 1 GHz, the results of analysis would be comparable with the full-wave analysis for frequencies around 1 GHz and if conductors are not rough. Though static solver would fail to predict behavior of the packaging interconnect, because of the loss extraction frequency 1 GHz is in the middle of the transition to skin-effect. **In both examples we neglected dielectric losses and dispersion related to polarization. The extraction results would be different if the dielectric loss and dispersion are accounted for.** In a follow-up paper we will investigate the effects of lossy dispersive dielectrics separately. Fortunately, it can be done with an electromagnetic solver for methodological purpose. **In reality all dielectric and conductor-related effects are not separable, especially at high frequencies.** Note that the only limitation of the used extraction technology is that it does not capture the first transition from DC to the state with current concentrated below the strip at low frequency band.
Conclusion

Only qualitative analysis of the existing conductor-effect simulation technologies has been provided in this paper. Though, we did multiple comparisons of results from different tools to draw some conclusions. The requirements of the ultra-broadband modeling of multi-gigabit channels push the limits of the computational electromagnetics. An ideal tool to account for all those effects has to satisfy the following requirements:

1. 3D full-wave analysis to extract parameters of transmission line and transitions
2. Simulate DC and low-frequency current crowding in planes
3. Simulate transition to skin-effect, shape and proximity effects at medium frequencies
4. Account for skin-effect, dispersion and edge effect at high frequencies
5. Have conductor models valid and causal over 5-6 frequency decades in general
6. Account for conductor surface roughness
7. Automatically extract S-parameters as well as frequency-dependent modal and RLGC matrix parameters per unit length for W-element models of multiconductor lines

Note, that there is no analysis tool that can satisfy all those requirements. Thus, tool selection for a particular problem requires understanding the limitations and benefits of the built-in in the simulator technology. For instance, the transmission line parameters extraction usually requires broadband and causal conductor models, but via-hole analysis may be done with simplified models and sometime even neglecting conductor loss at all. This is because of relatively small electrical length of via and dominance of the reflection and radiation losses. Being just 3D and full-wave does not automatically mean the highest accuracy possible and sometime even a 2D static field solver may provide more accurate solutions than a 3D full-wave solver without the broadband conductor models.

References

5. Simbeor - 3D full-wave field solver for multilayered circuits, Simberian Inc., [www.simberian.com](http://www.simberian.com).

Solutions generated to illustrate this paper are available at [http://www.simberian.com/AppNotes/Solutions/ModelingConductorLoss_2007_02.zip](http://www.simberian.com/AppNotes/Solutions/ModelingConductorLoss_2007_02.zip)