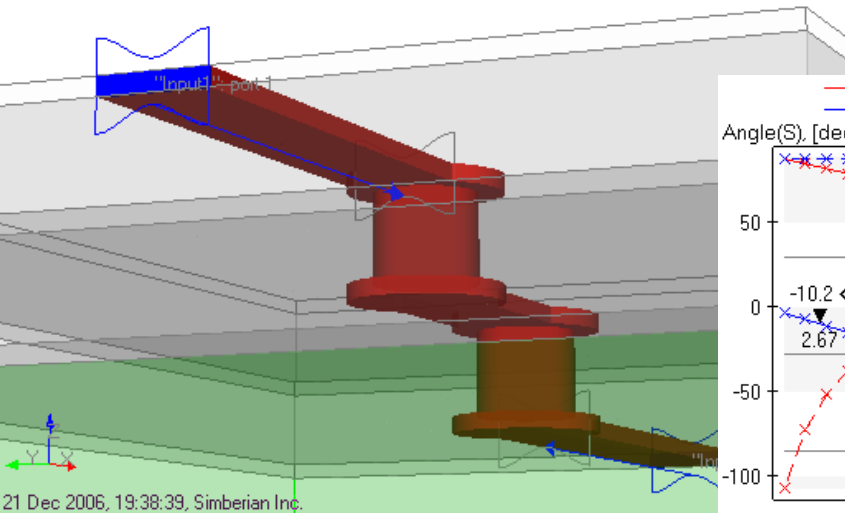


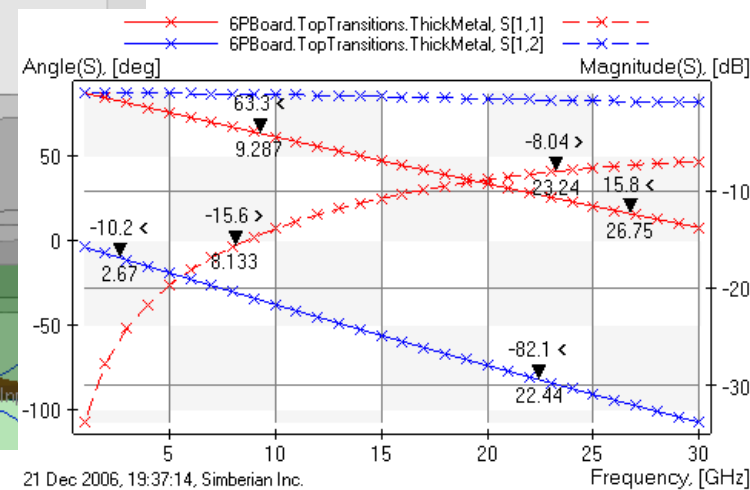
Sensitivity of PCB Material Identification with GMS-Parameters to Variations in Test Fixtures

Solution: "MicroVias"

- 6PBoard
 - Materials
 - "copper", RRes=1, Rough=0.01
 - "IdealMetal"
 - "prepreg", DK=4.7, LT=c
 - "Vacuum"
 - "FR4", DK=4.2, LT=0.02
 - StackUp: LU=[mil], NL=15, T
 - TopTransitions
 - CircuitData: LU=[mil]
 - Multiport: 2 inputs, 2
 - LatticeBox
 - Geometry
 - GeoComposite: "
 - TLines
 - Inputs
 - ThickMetal
 - CollapsedMetal
 - BottomTransition



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Overview

- Introduction
- Identification of material parameters with Generalized Modal S-parameters
- Sensitivity of GMS-parameters to non-identity of cross-sections
- Sensitivity of GMS-parameters to non-identity of launches
- Effect of bends on GMS-parameters
- Conclusion

Introduction

- Broadband dielectric and conductor models are the requisite foundation for performing meaningful electromagnetic verification of multi-gigabit interconnects
- Such model can be effectively identified with Generalized Modal S-parameters (GMS-parameters)
 - The method is the simplest possible and is based on fitting computed and measured GMS-parameters as outlined in:
Y. Shlepnev, A. Neves, T. Dagostino, S. McMorrow, Practical identification of dispersive dielectric models with generalized modal S-parameters for analysis of interconnects in 6-100 Gb/s applications – DesignCon 2010 – available at <http://www.designcon.com/infovault/>
- Measured S-parameters of 2 line segments are required to compute GMS-parameters of line difference
- Transmission lines in both segments should have substantially identical cross-sections and connector or probe launches
- In reality, both cross-sections and transitions are not identical and test fixture may have discontinuities such as bends and vias
- This app note investigates sensitivity of the GMS-based method to variations in geometry of cross-sections and launches and presence of discontinuities in test fixtures
- Simbeor 2011 (64bit) built on Nov. 15th 2010 is used to generate the results

Overview

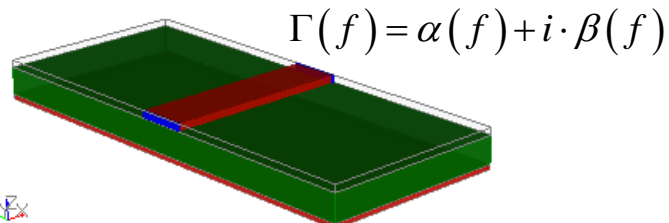
- Introduction
- Identification of material parameters with Generalized Modal S-parameters (GMS)
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Material parameters identification with GMS-parameters

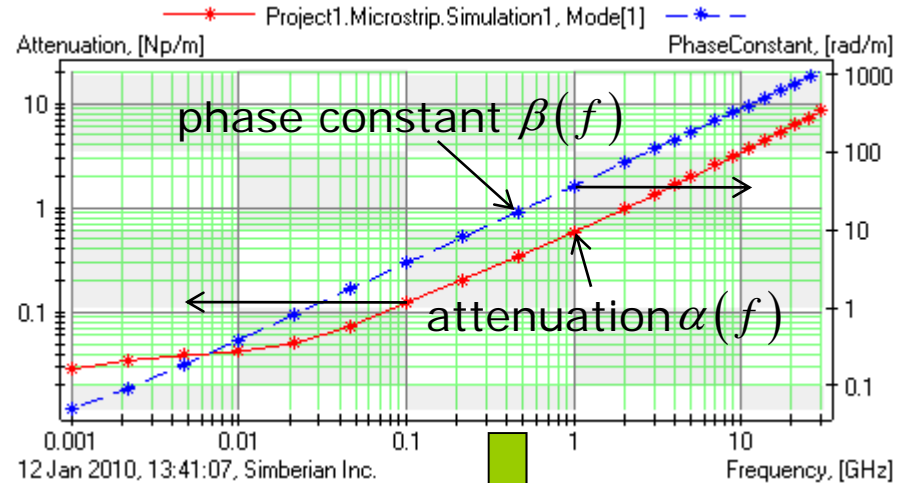
- ❑ Measure S-parameters of two test fixtures with different length of line segments S1 and S2
- ❑ Transform S1 and S2 to the T-matrices T1 and T2, diagonalize the product of T1 and inversed T2 and compute GMS-parameters of the line difference
- ❑ Select material model and guess values of the model parameters
- ❑ Compute GMS-parameters of the line difference segment by solving Maxwell's equation for t-line cross-section (only propagation constants are needed)
- ❑ Adjust material parameters until computed GMS parameters fit measured GMS-parameters with the computed

Generalized Modal S-parameters (GMS-parameters) for one-conductor line

1. Compute propagation constant (Gamma)



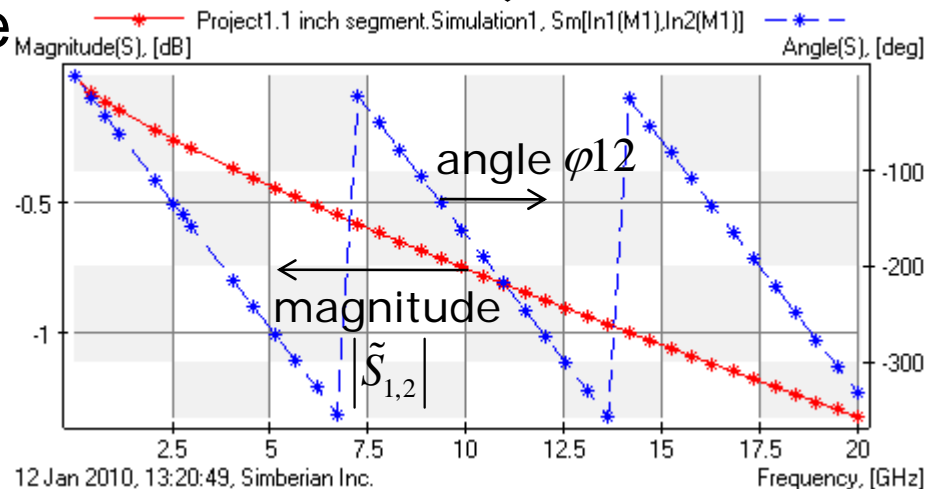
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2. Compute 2x2 GMS of line segment with length dL

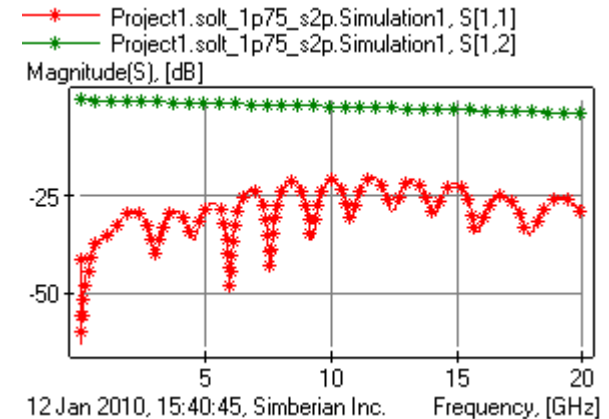
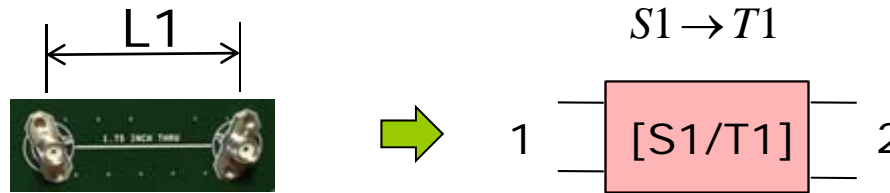
$$GMSc = \begin{bmatrix} 0 & \exp(-\Gamma \cdot dL) \\ \exp(-\Gamma \cdot dL) & 0 \end{bmatrix}$$

Very simple!

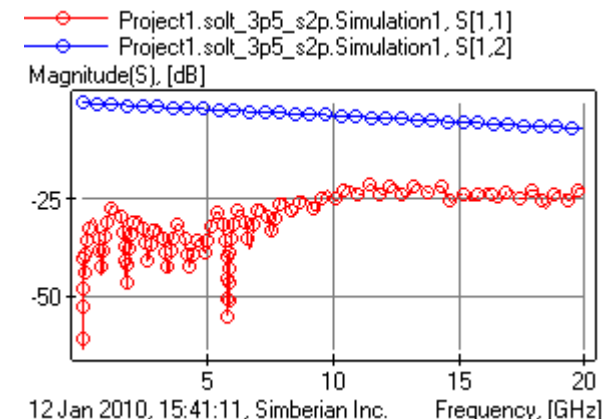
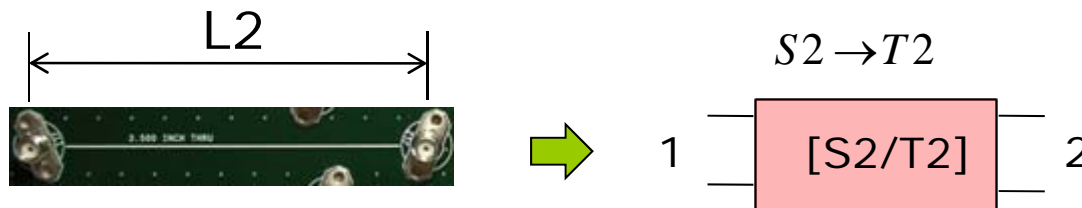


Measure S-parameters of two test fixtures with line segments (no calibration is required)

□ S1 and T1 for line with length L1



□ S2 and T2 for line with length L2

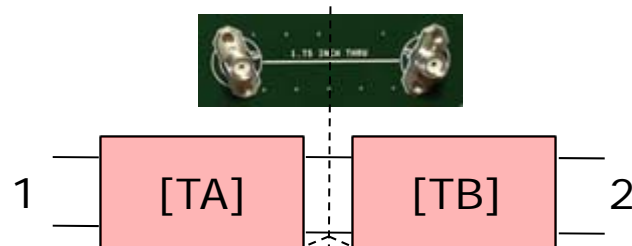
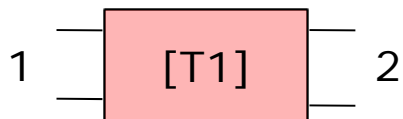


T1 and T2 matrices are scattering T-parameters (computed directly from S-parameters)

Extract Generalized Modal T-parameters (GMT) and then GMS-Parameters

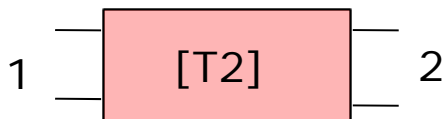
Segment L1

$$T1 = TA \cdot TB$$



Segment L2

$$T2 = TA \cdot GMT \cdot TB$$



GMT is non-reflective modal T-matrix (normalized to the unknown characteristic impedances of the modes)

$$T2 \cdot T1^{-1} = TA \cdot GMT \cdot TA^{-1}$$



$$GMT \doteq \text{eigenvals}(T2 \cdot T1^{-1})$$

Easy to compute!

For 1-conductor line we get:

$$GMT = \begin{bmatrix} T_{11} & 0 \\ 0 & T_{11}^{-1} \end{bmatrix}$$



$$GMSm = \begin{bmatrix} 0 & T_{11} \\ T_{11} & 0 \end{bmatrix}$$

Just 1 complex function!

Identifying dielectrics by fitting GMS-parameters

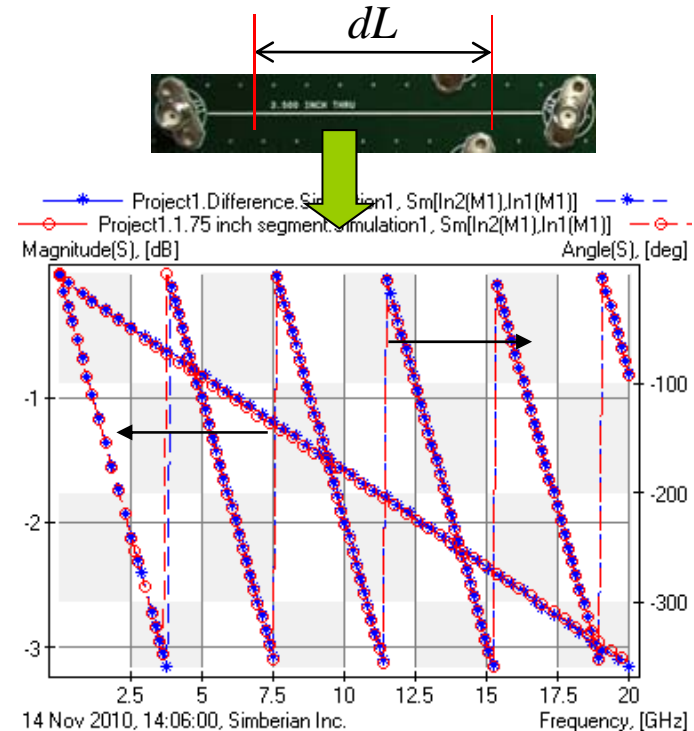
- Solve Maxwell's equations for 1-conductor line:

$$GMS_c = \begin{bmatrix} 0 & \exp(-\Gamma \cdot dL) \\ \exp(-\Gamma \cdot dL) & 0 \end{bmatrix}$$

- Fit measured data:  *Only 1 complex function!*

$$GMS_m = \begin{bmatrix} 0 & T_{11} \\ T_{11} & 0 \end{bmatrix}$$

- Measured GMS-parameters of the segment can be directly fitted with the calculated GMS-parameters for material parameters identification
- **Phase or group delay can be used to identify DK and insertion loss to identify LT or conductor roughness!**



The GMS-parameters technique is the simplest possible

- Needs un-calibrated measurements for 2 t-lines with any geometry of cross-section and transitions
 - No extraction of propagation constants (Γ) from measured data (difficult, error-prone)
 - No de-embedding of connectors and launches (difficult, error-prone)
- Needs the simplest numerical model
 - Requires computation of only propagation constants
 - No 3D electromagnetic models of the transitions
- Minimal number of smooth complex functions to match
 - One parameter for single and two parameters for differential
 - All reflection and modal transformation parameters are exactly zeros

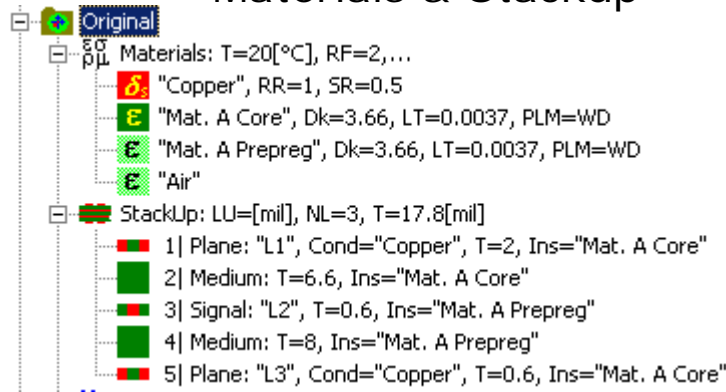
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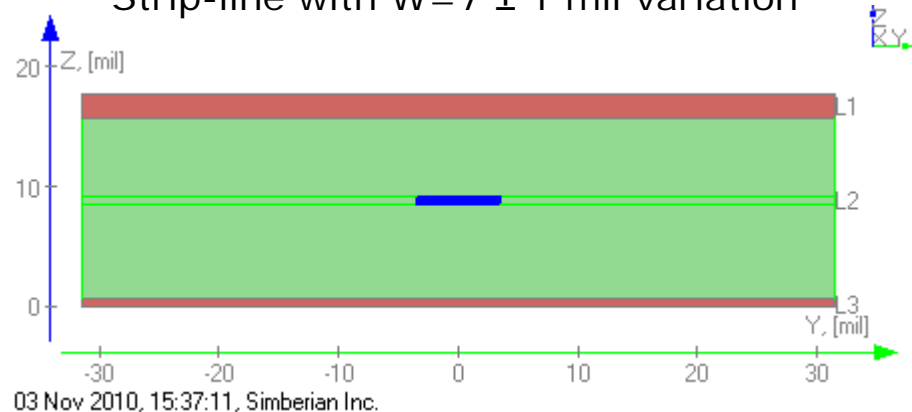
What if 2 lines used for identification have non-identical cross-sections?

- Numerical experiment to investigate the consequences of the non-identity

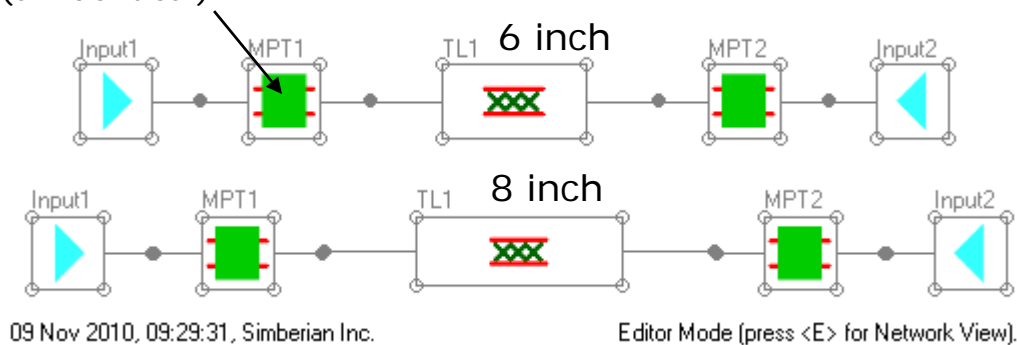
Materials & Stackup



Strip-line with $W=7 \pm 1$ mil variation



Models of the launches (all identical)



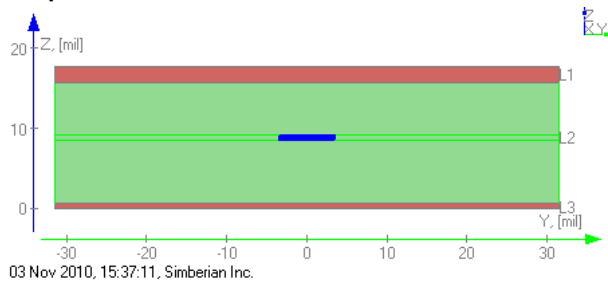
09 Nov 2010, 09:29:31, Simberian Inc.

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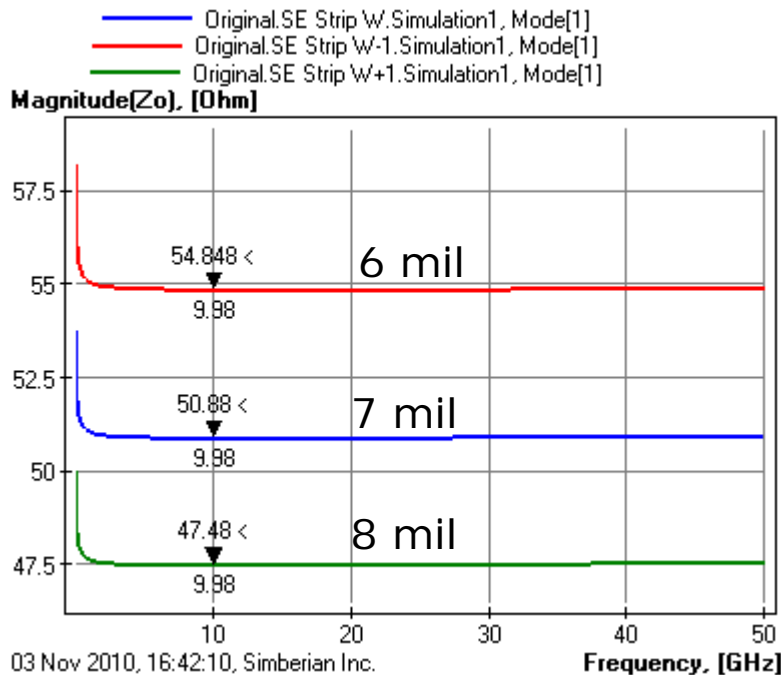
From simulated S-parameters of 2 structures with varying strip widths we extract GMS-parameters of 2-inch segment and compare it with the GMS-parameters of 2-inch segment computed directly

T-line parameters

Strip-line with $W=7 \pm 1$ mil variation

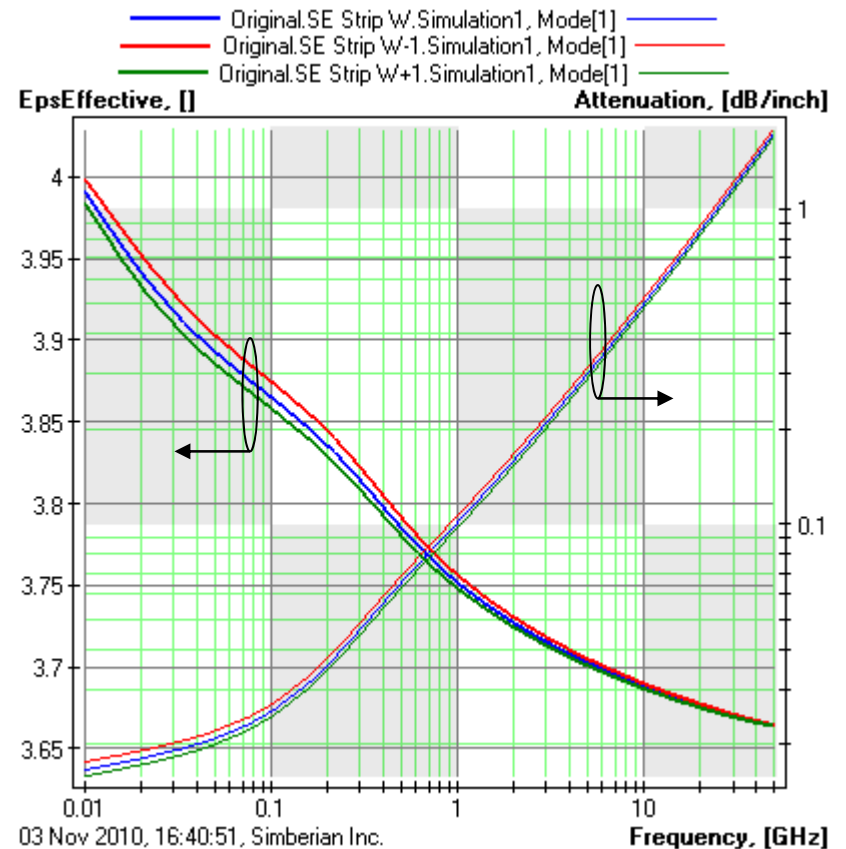


03 Nov 2010, 15:37:11, Simberian Inc.



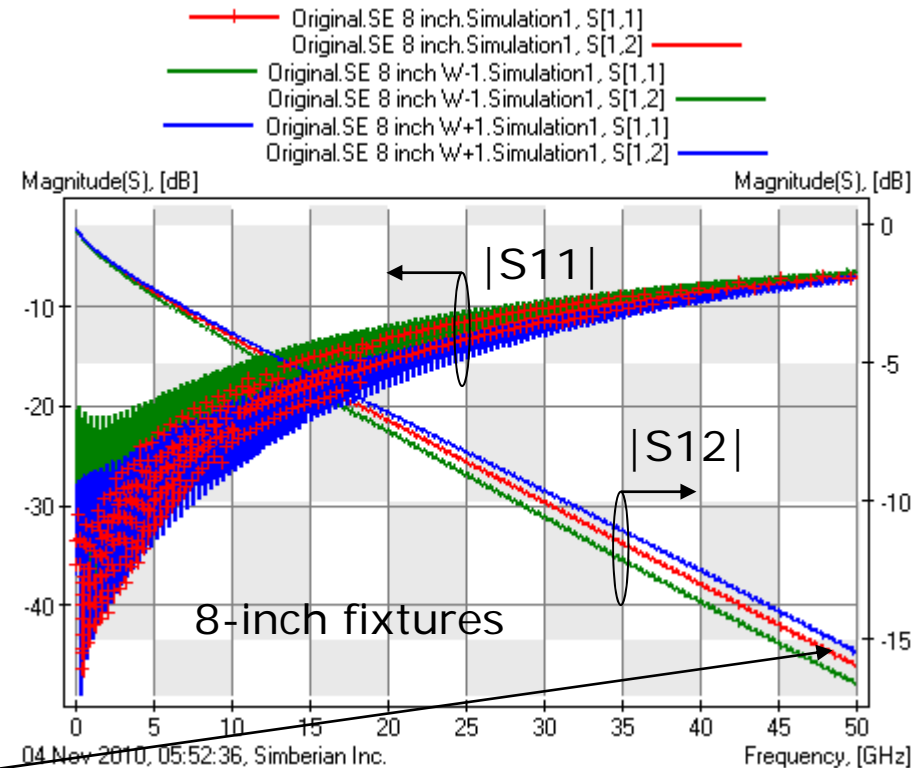
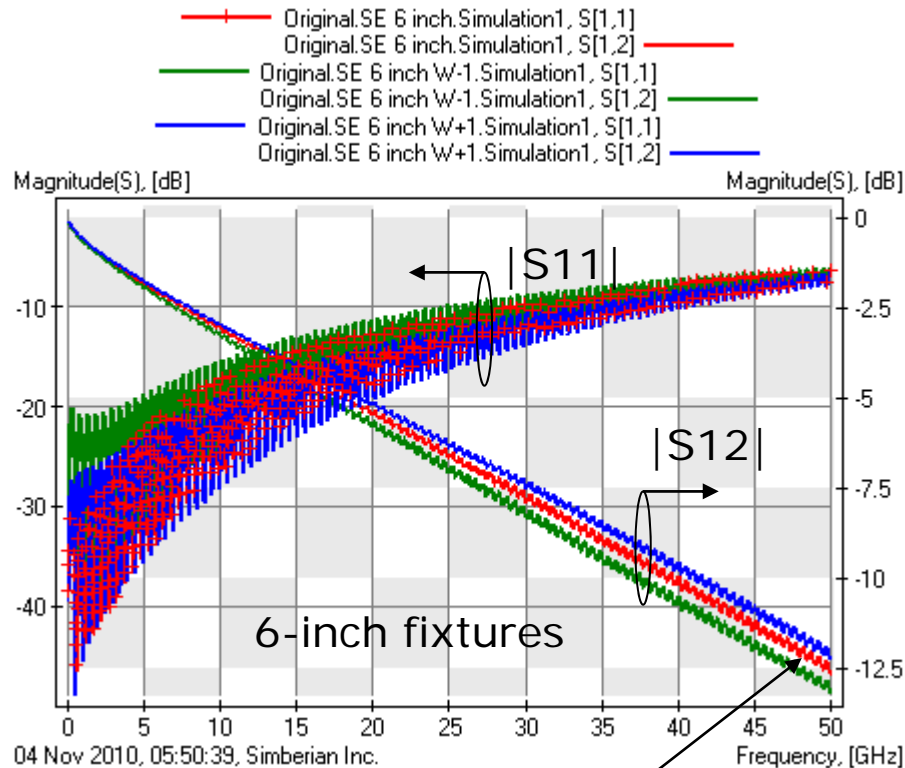
03 Nov 2010, 16:42:10, Simberian Inc.

Large variation of Z_0 and smaller variation of propagation constant parameters



03 Nov 2010, 16:40:51, Simberian Inc.

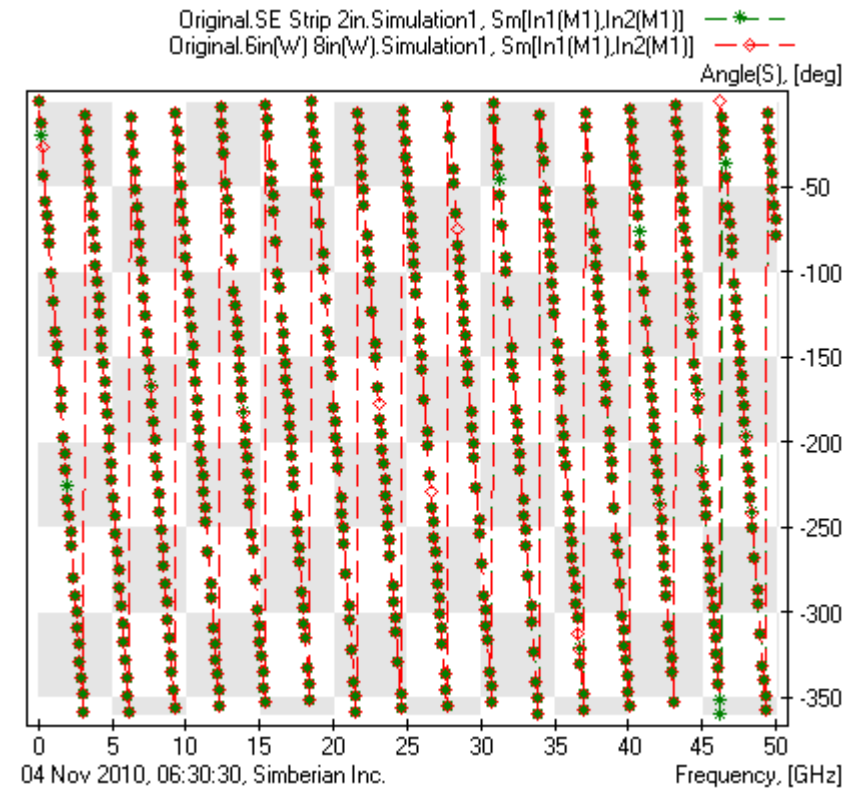
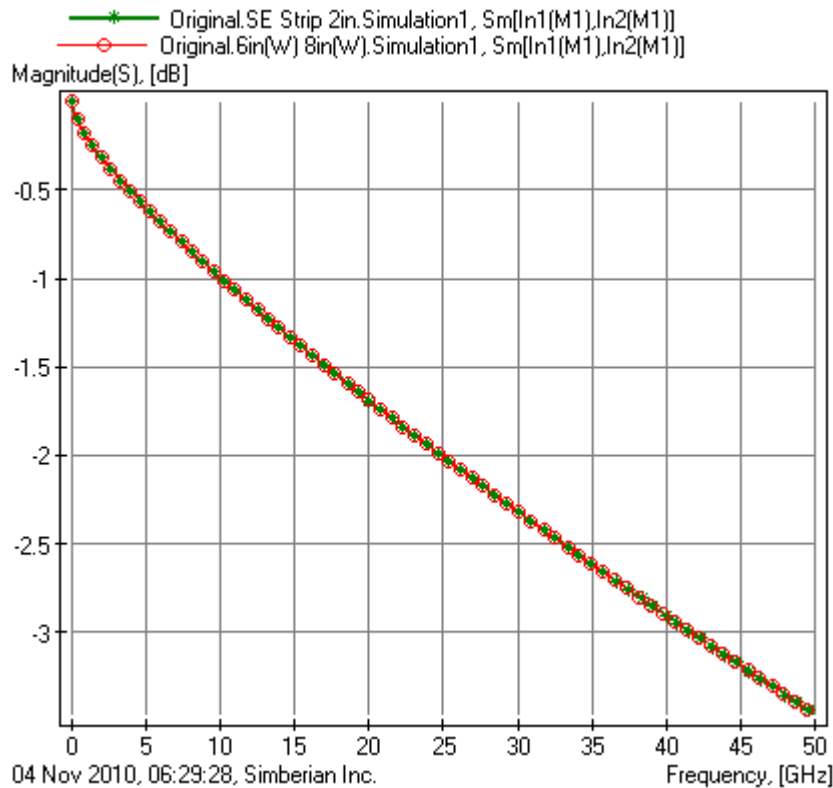
Effect of strip width on S-parameters of the test fixtures



$W_{\pm 1}$ gives about ± 0.5 dB difference in transmission coefficient at 50 GHz
 Phases are almost the same
 Reflection is mostly due to the reflection at the launches

Identical strip widths in test fixtures

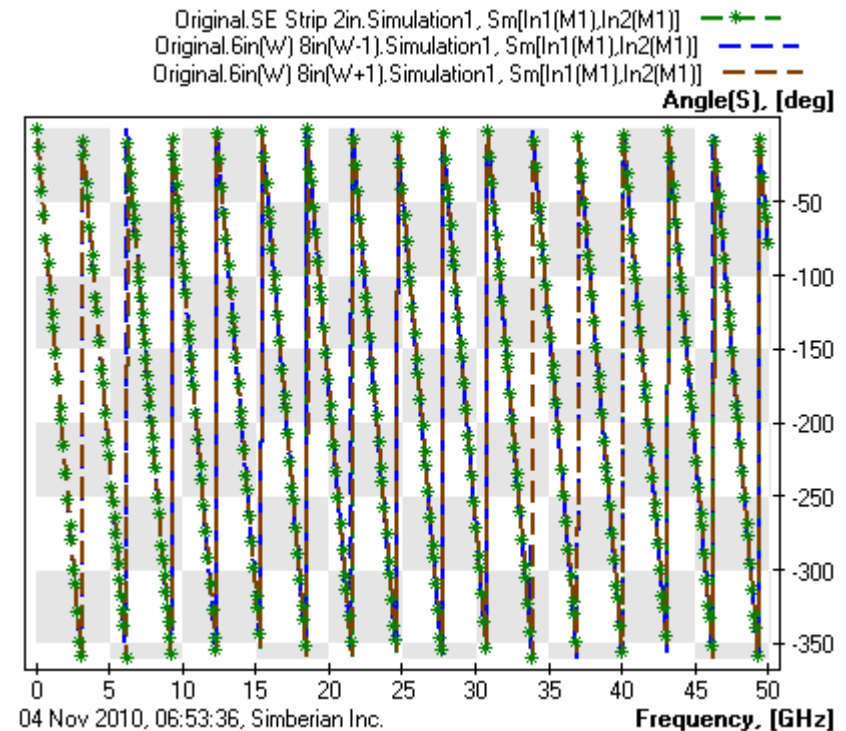
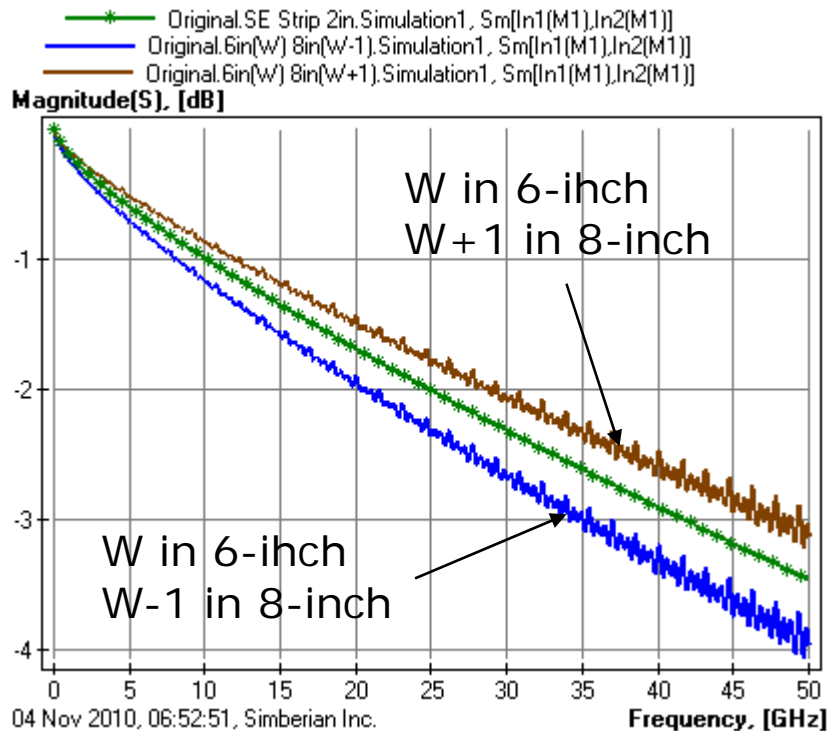
GM transmission of 2-inch segment (green stars) match GM transmission extracted from S-parameters of 2 test fixtures (red circles)



The result is independent of actual width and launch construction as long as the widths and launches are identical in the test fixtures!

1 mil width difference in test fixtures

Magnitude of GM transmission of 2-inch segment (green stars) do not match GM transmission extracted from S-parameters of 2 test fixtures (brown and blue)

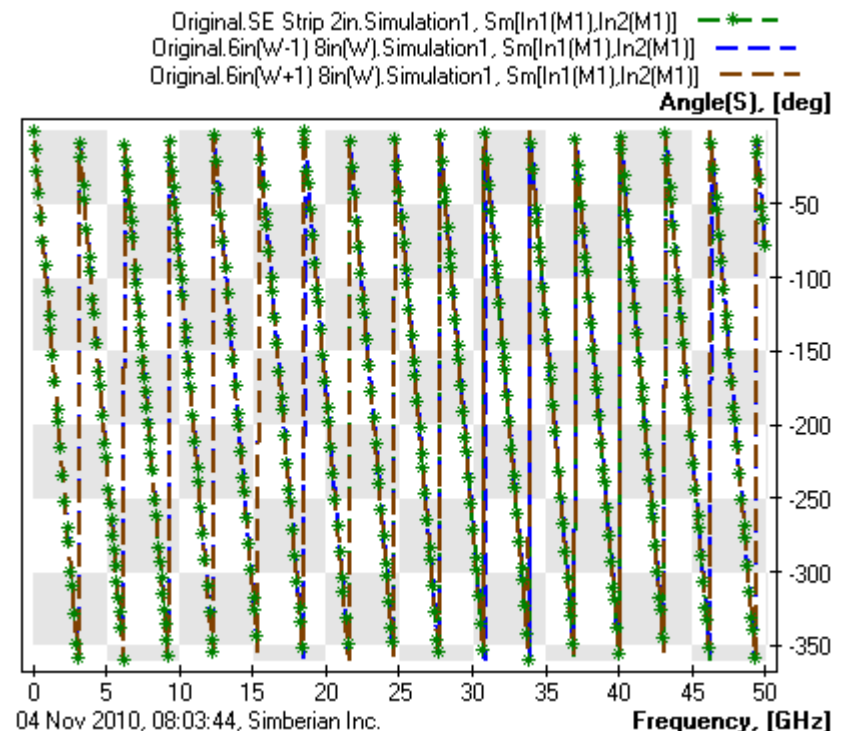
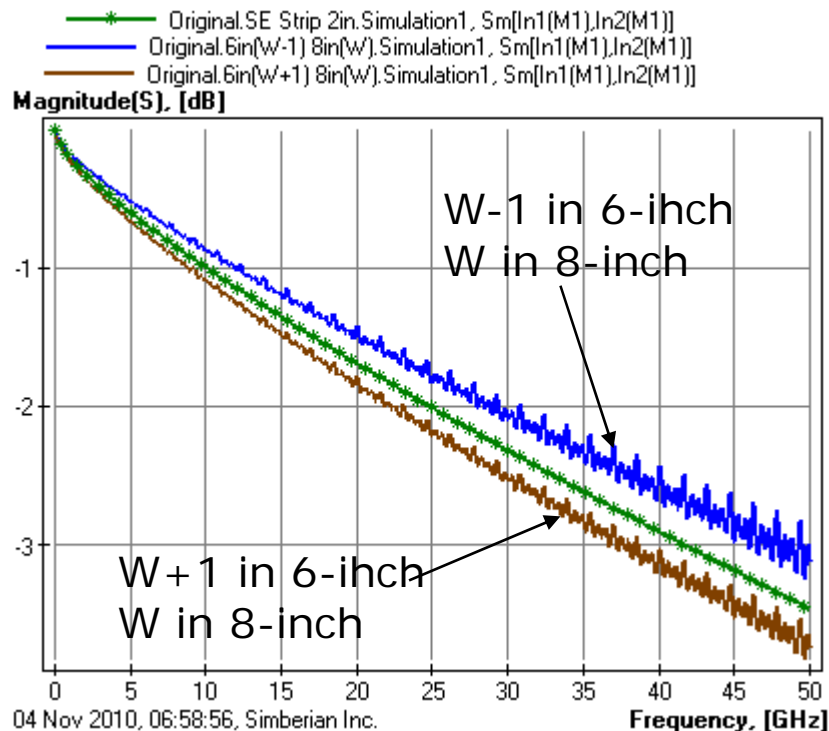


Smaller width in longer test fixture -> larger insertion loss
Larger width in longer test fixture -> smaller insertion loss
Identical phases – same identified DK!

1 mil width difference in test fixtures

Magnitude of GM transmission of 2-inch segment (green stars) do not match GM transmission extracted from S-parameters of 2 test fixtures (brown and blue)

Less sensitive to variations in shorter test fixture



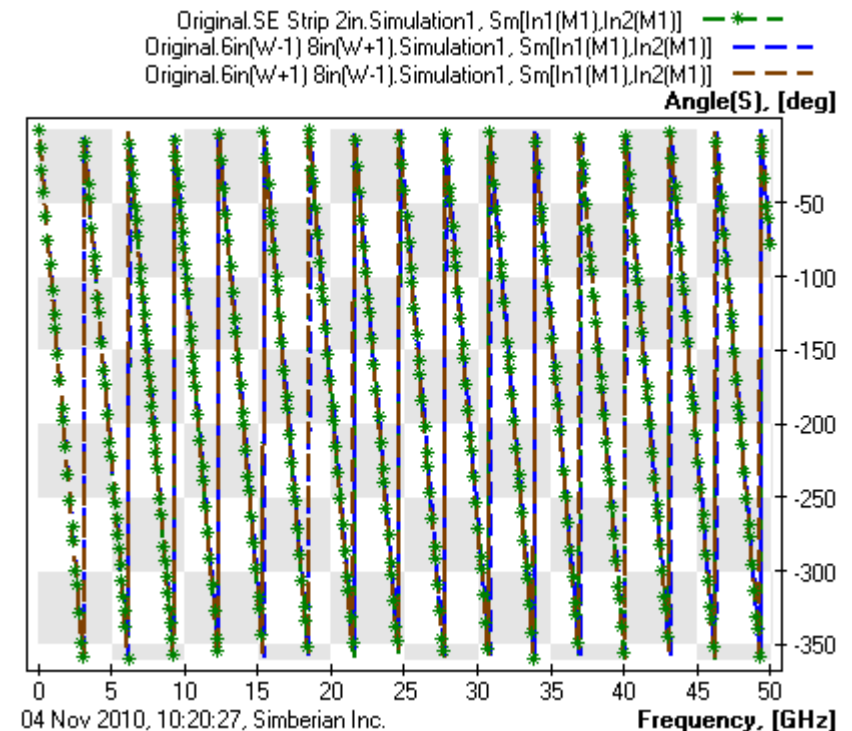
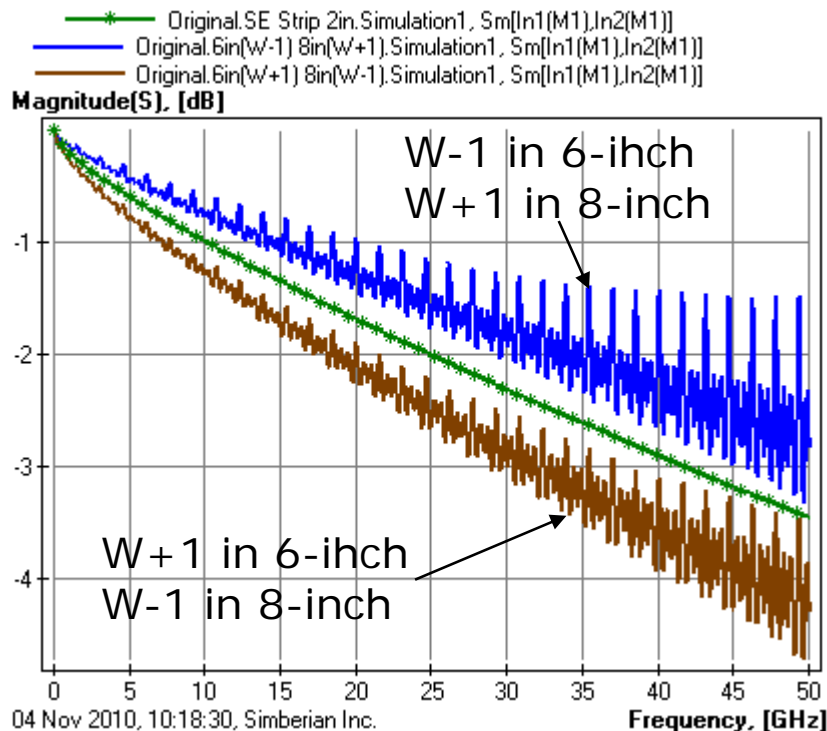
Smaller width in shorter test fixture -> smaller insertion loss

Larger width in shorter test fixture -> larger insertion loss

Identical phases – same identified DK!

2 mil width difference in test fixtures

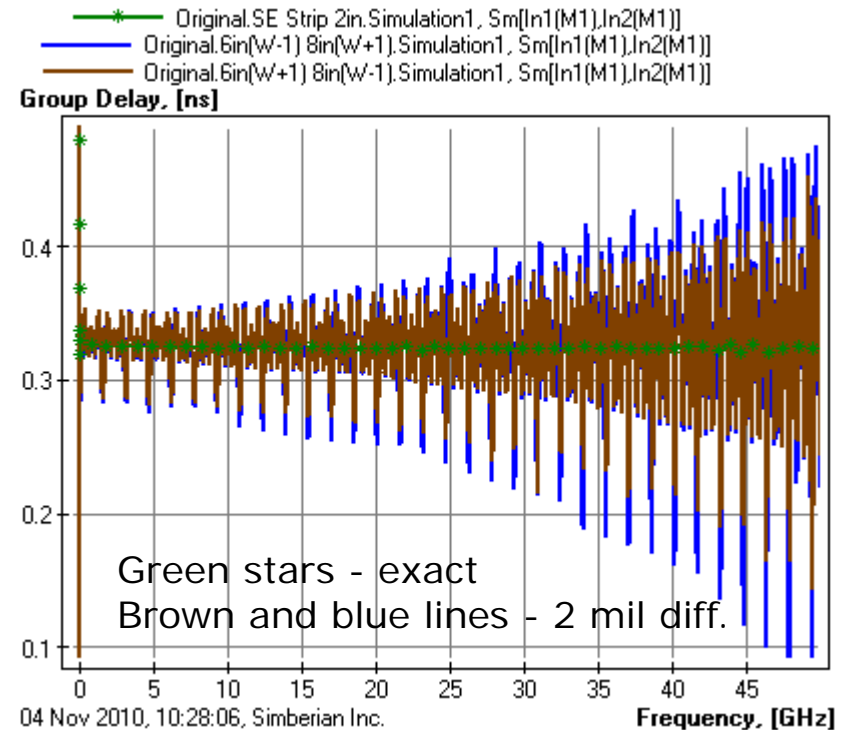
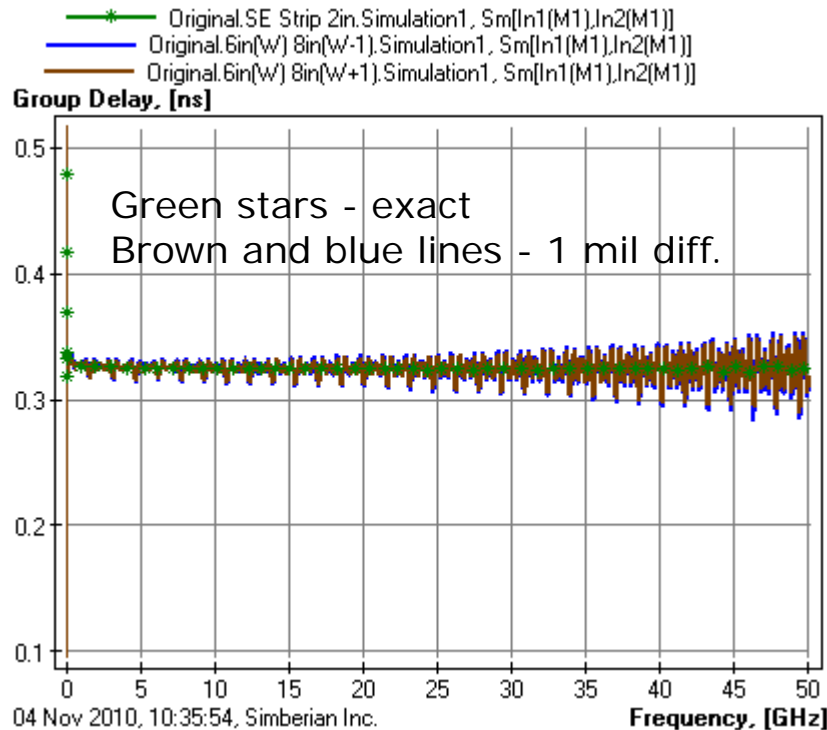
Magnitude of GM transmission of 2-inch segment (green stars) do not match GM transmission extracted from S-parameters of 2 test fixtures (brown and blue)



Larger width in longer test fixture -> smaller insertion loss
 Smaller width in longer test fixture -> larger insertion loss
 Identical phases – same identified DK!

Effect of width variation on group delay

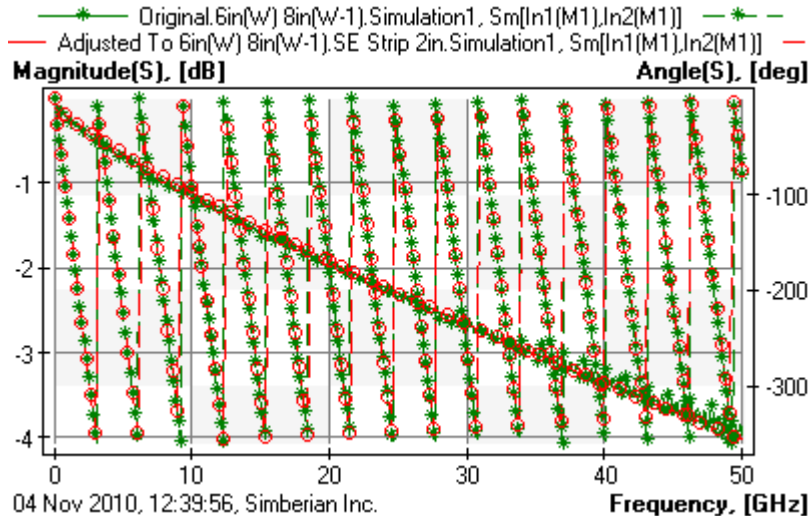
Magnitude of GM transmission of 2-inch segment (green stars) do not match GM transmission extracted from S-parameters of 2 test fixtures (brown and blue)



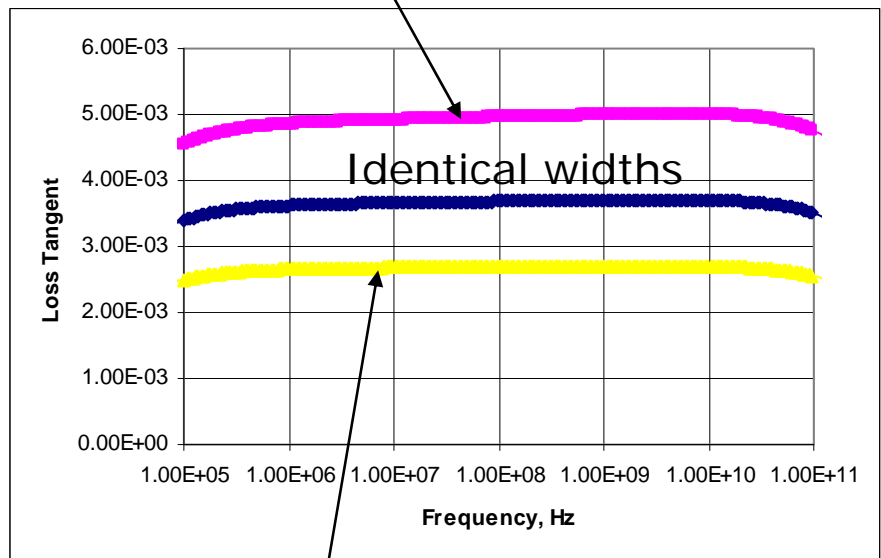
Group delay becomes not usable for DK identification with large strip width variations!

What if we use test fixture with variations of W for identification of dielectric model?

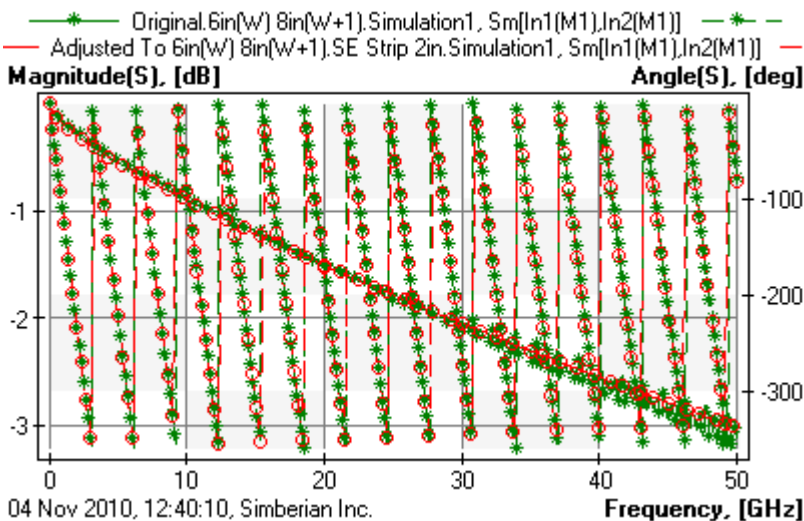
Wideband Debye model is used here



7 mil in 6-inch and 6 mil in 8-inch:
 ~10% larger identified LT



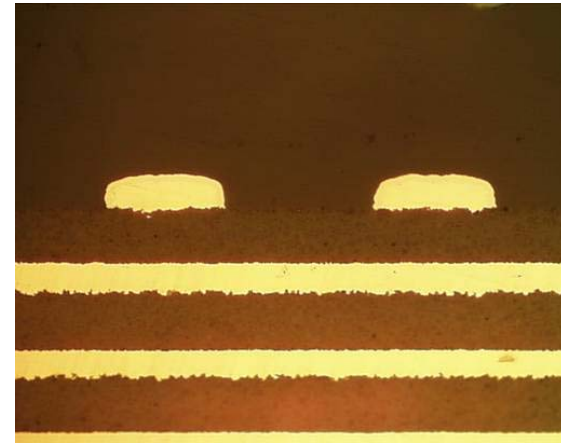
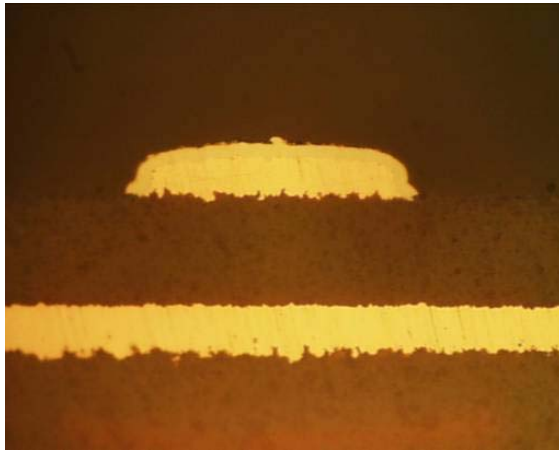
7 mil in 6-inch and 8 mil in 8-inch:
 ~7% smaller identified LT



Identified DK is identical in all cases
 (almost same transmission phases)

Difference in cross-section shape will have similar effect on loss identification

- Real cross-sections may be not rectangular or trapezoidal!



- Difference in current distribution in simulated rectangular or trapezoidal conductor will produce different loss
 - This is the major source of errors in identification of loss parameters in case of low-loss dielectrics at high frequencies
 - **Not a problem in case of regular FR-4 with $LT \sim 0.02$**

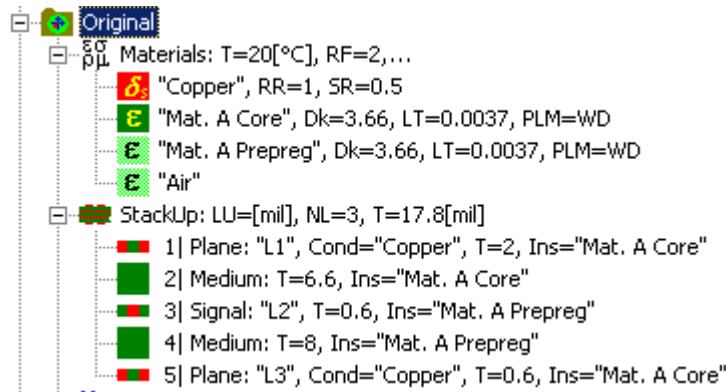
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What if launches or connectors in test fixtures are not identical?

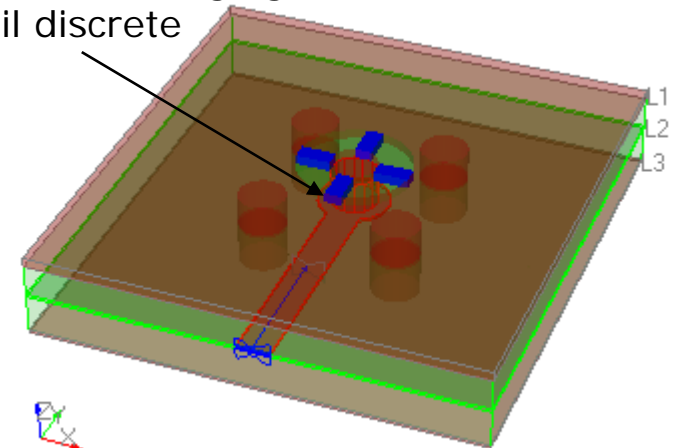
- Numerical experiment to investigate the consequences of the non-identity

Materials & Stackup

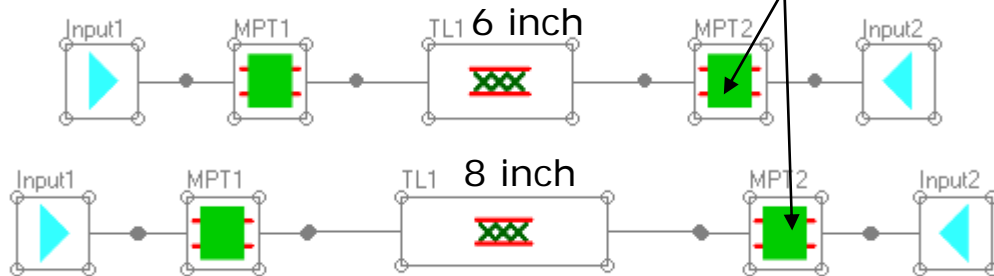


Simple transition to 7-mil strip-line with pad (in L2) diameter changing from 8 to 22 mil with 3.5 mil discrete

- T0 – 8 mil
- T1 – 11.5 mil
- T2 – 15 mil
- T3 – 18.5 mil
- T4 – 22 mil



Models of the launches – different between 2 structures



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From simulated S-parameters of 2 structures with varying pad diameters we extract GMS-parameters of 2-inch segment and compare it with the GMS-parameters of 2-inch segment computed directly

S-parameters of the launches

- The larger the diameter of the pad, the larger the reflection $|S_{11}|$ and the smaller the transmission $|S_{12}|$ parameter

Pad diameter:

T0 – 8 mil

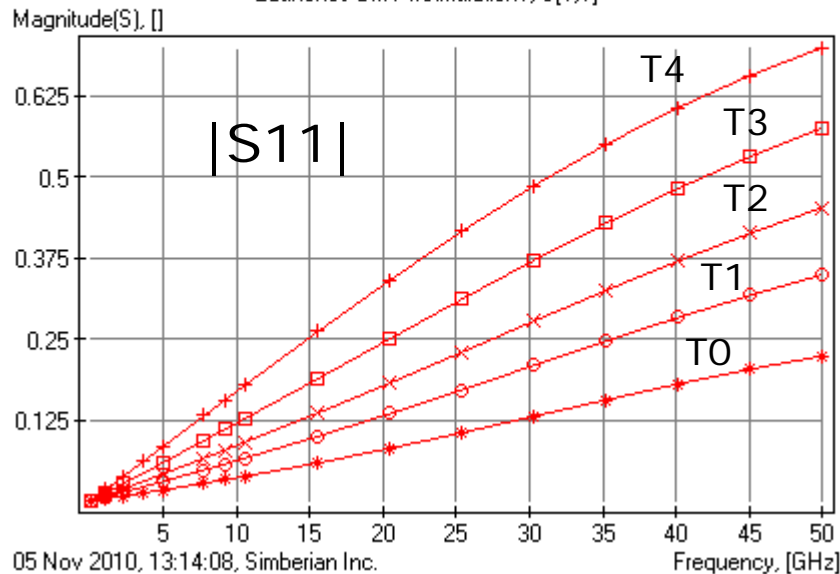
T1 – 11.5 mil

T2 – 15 mil

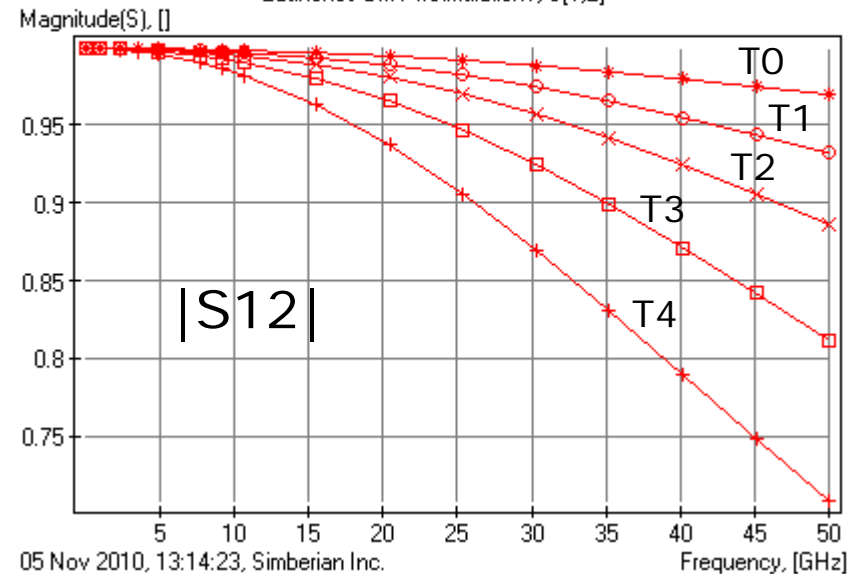
T3 – 18.5 mil

T4 – 22 mil

- * Launches Off.T0.Simulation1, S[1,1]
- Launches Off.T1.Simulation1, S[1,1]
- × Launches Off.T2.Simulation1, S[1,1]
- Launches Off.T3.Simulation1, S[1,1]
- + Launches Off.T4.Simulation1, S[1,1]

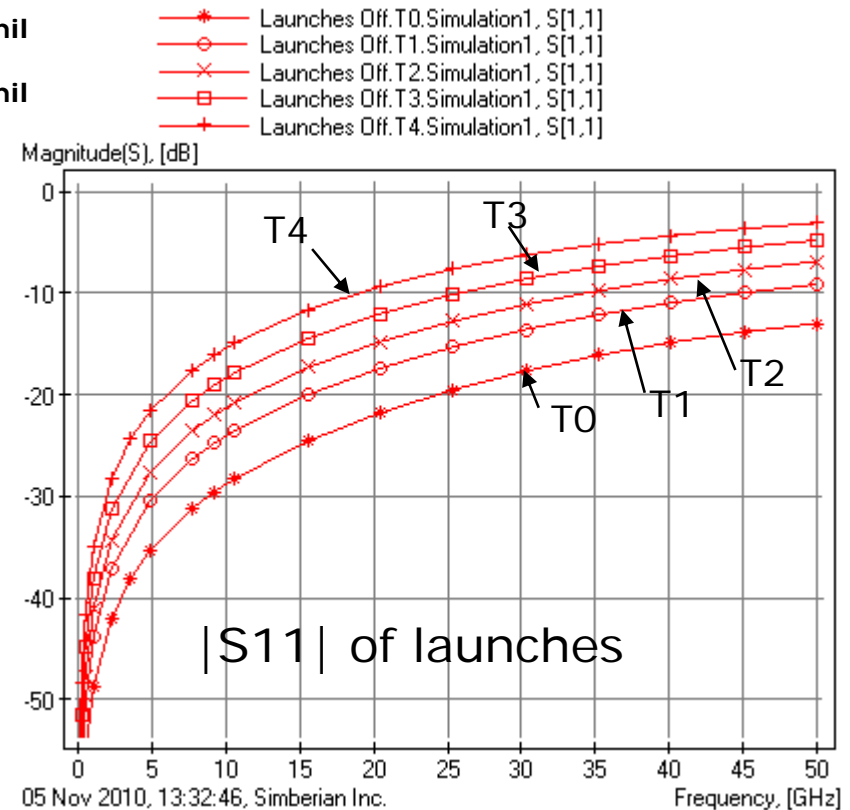
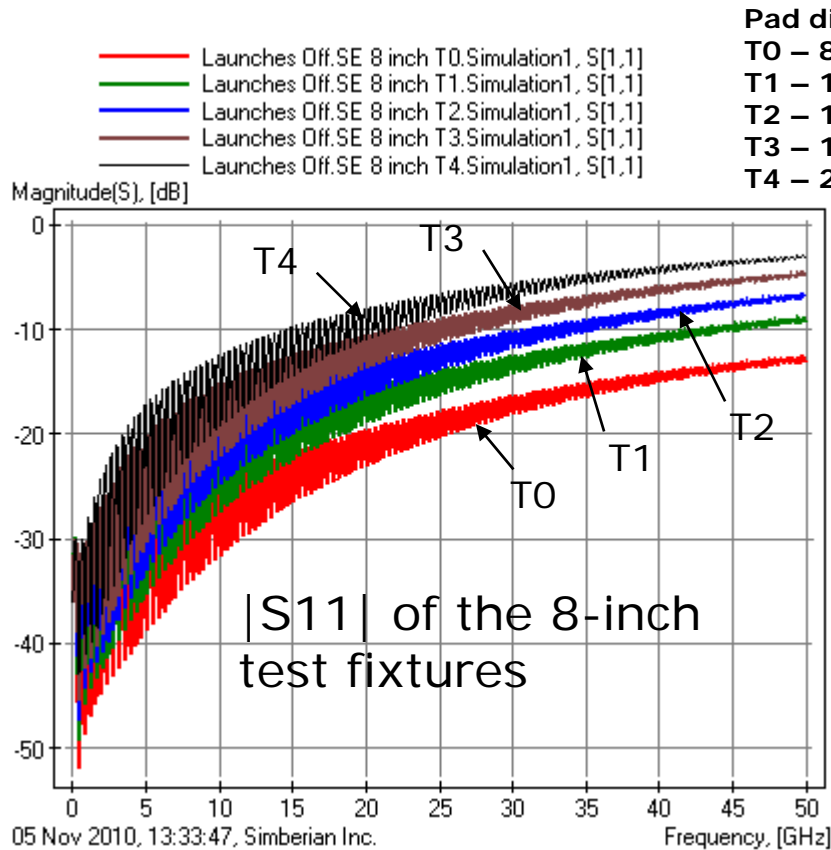


- * Launches Off.T0.Simulation1, S[1,2]
- Launches Off.T1.Simulation1, S[1,2]
- × Launches Off.T2.Simulation1, S[1,2]
- Launches Off.T3.Simulation1, S[1,2]
- + Launches Off.T4.Simulation1, S[1,2]



Effect of launch pad diameter on reflection from 8-inch test fixture

- In case of the t-line impedance close to 50-Ohm, the envelop of the reflection parameters is mostly defined by the reflection from the transition

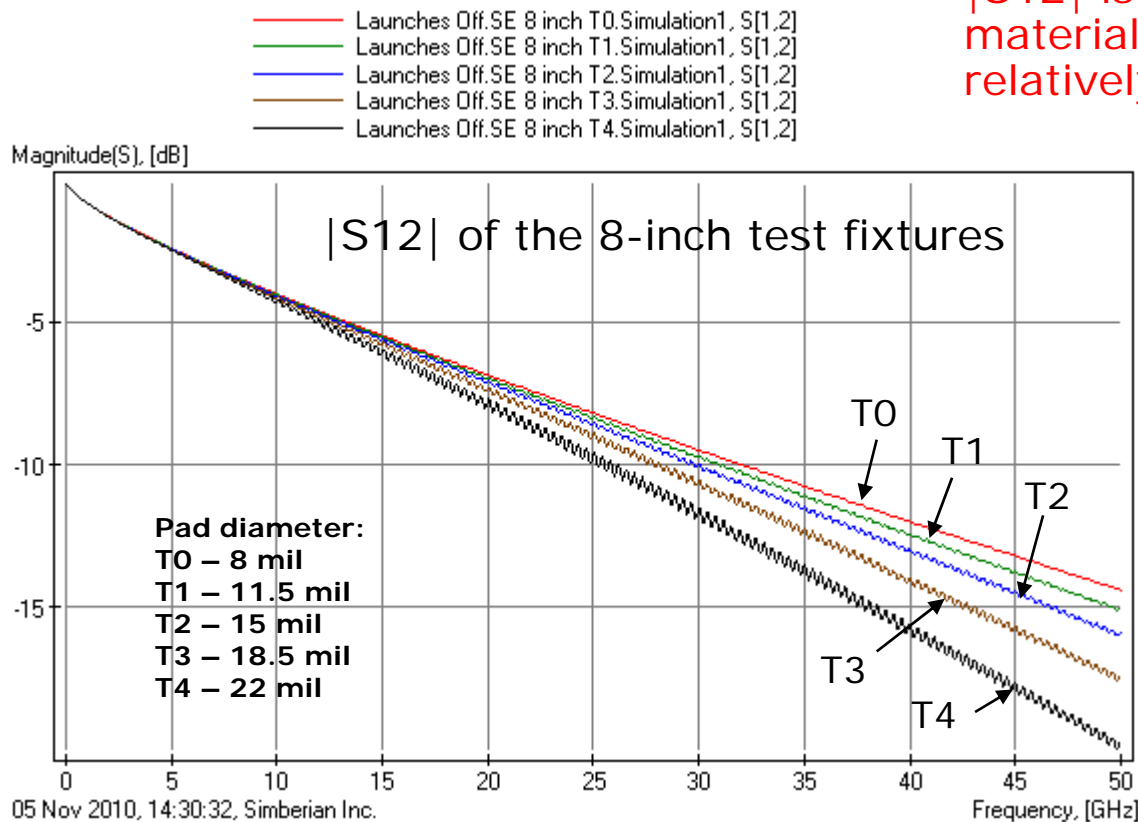


Behavior of 6-inch fixture is similar

Effect of launch pad diameter on transmission through 8-inch test fixture

- Reflective launch lead to substantial difference in the insertion loss $|S_{12}|$ of the test fixture

$|S_{12}|$ is not suitable for the material identification, even with relatively good launches!



Phases are practically identical

Group delays are substantially different due to reflections

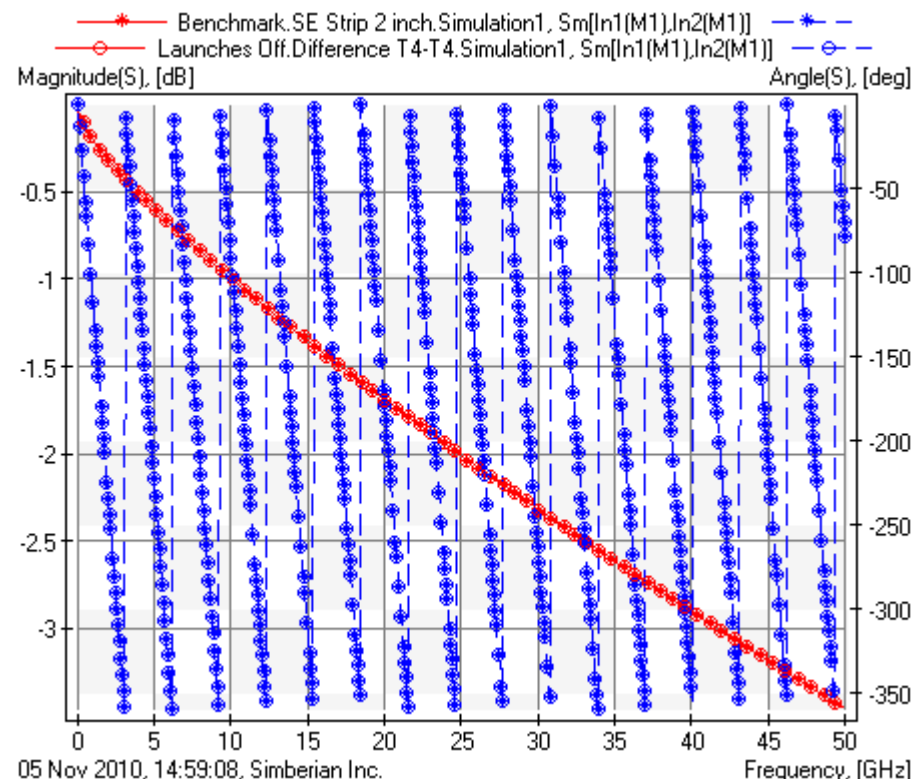
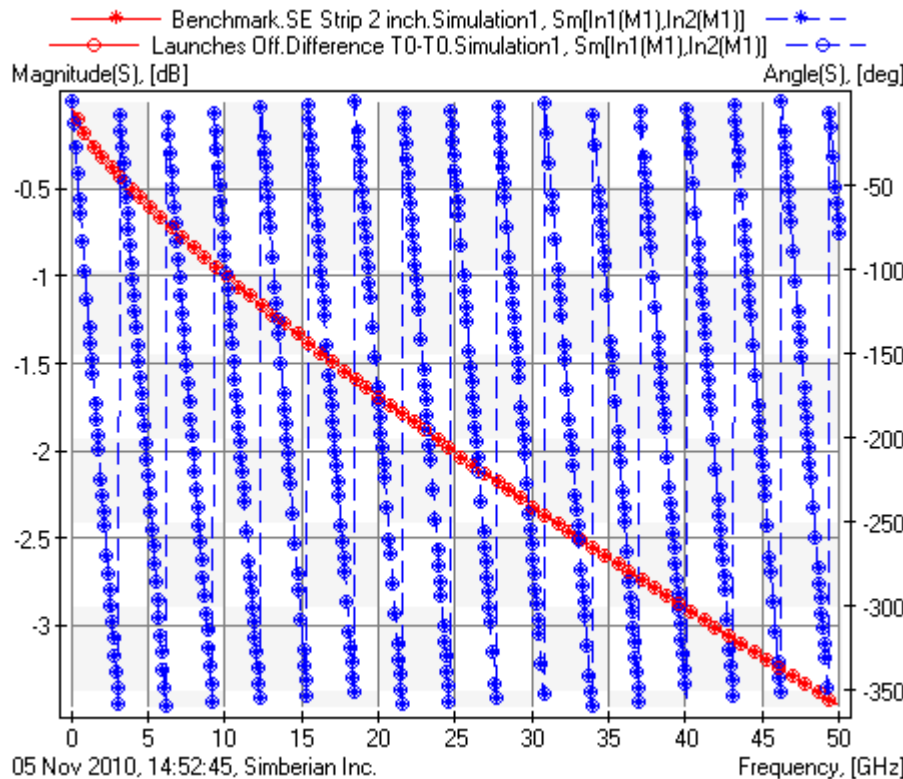
The result is similar for the 6-inch structure

GMS-parameters in case of identical launches

- Extracted GM transmission parameters of 2-inch segment are independent of the launch geometry as long as all 4 launches on 2 test fixtures are identical

Launch with 8 mil pad:

Launch with 22 mil pad:

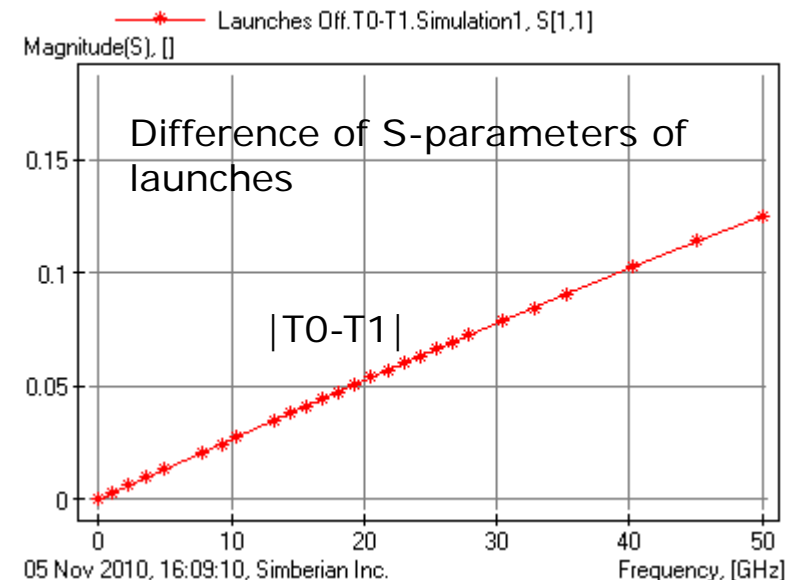
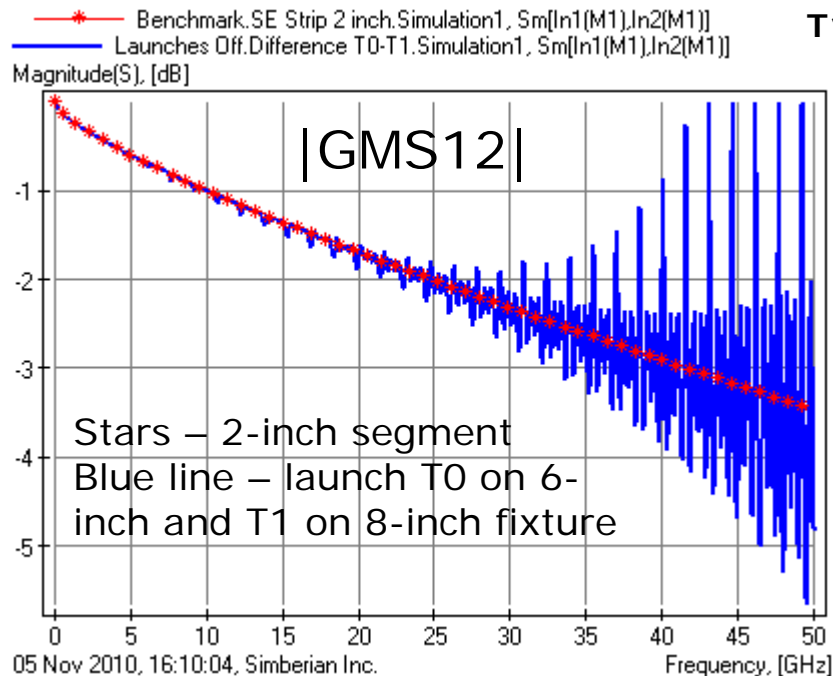


Stars – 2-inch exact, circles – computed from 2 test fixtures

What if launches on 6-inch fixture are different from launches on 8-inch fixture?

- Magnitude of Generalized Modal transmission looks “noisy”
- Material identification may be possible only up to 20-25 GHz

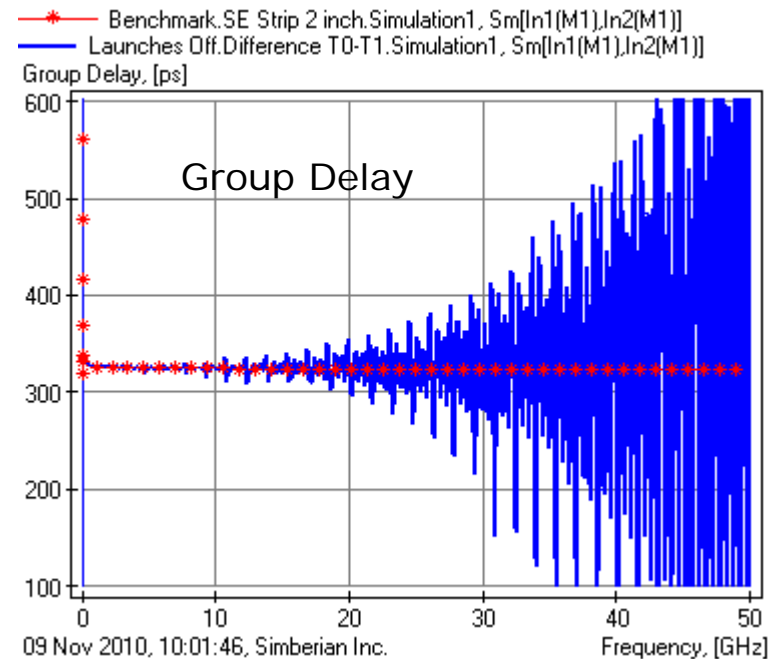
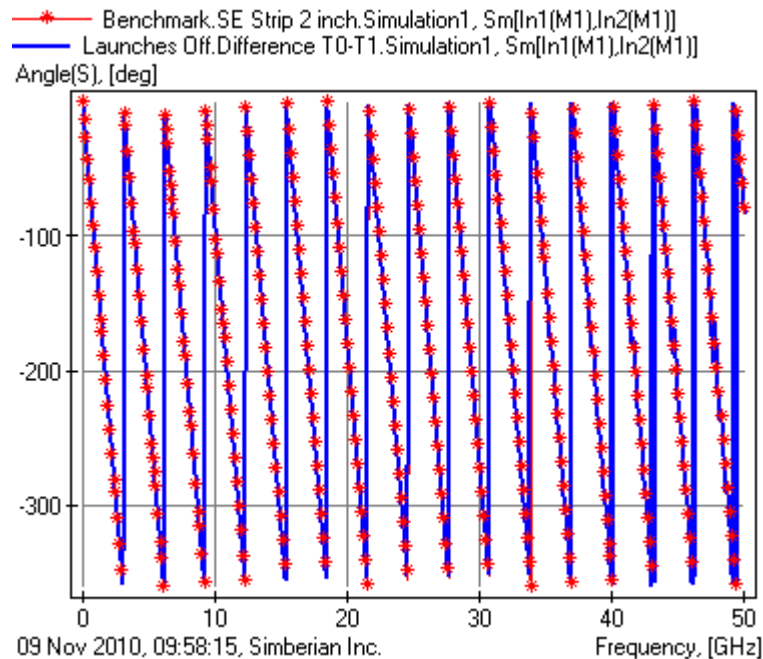
Pad diameter:
T0 – 8 mil
T1 – 11.5 mil



What if launches on 6-inch fixture are different from launches on 8-inch fixture?

- Phase of Generalized Modal transmission looks OK up to 40 GHz
- Group Delay is “noisy” starting from about 10 GHz

Arg(GMS12)



Stars – 2-inch segment
Blue line – launch T0 on 6-inch and T1 on 8-inch fixture

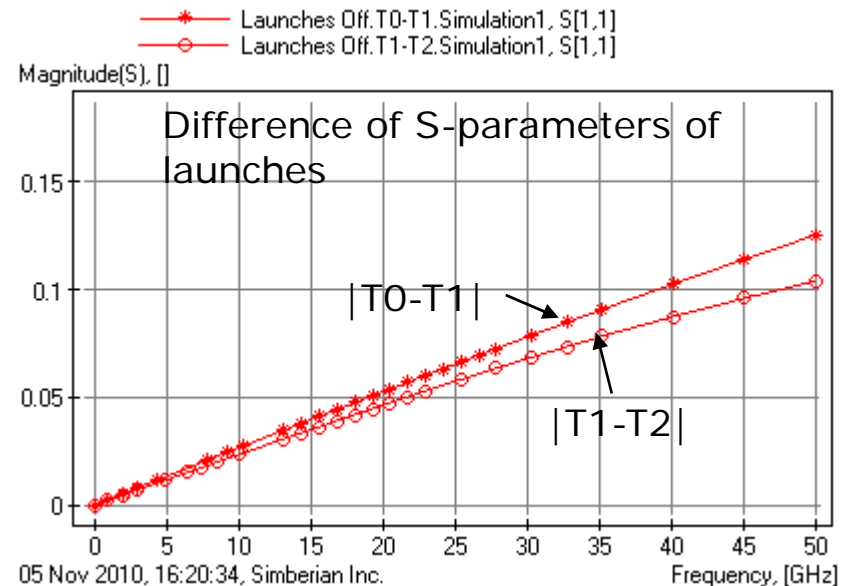
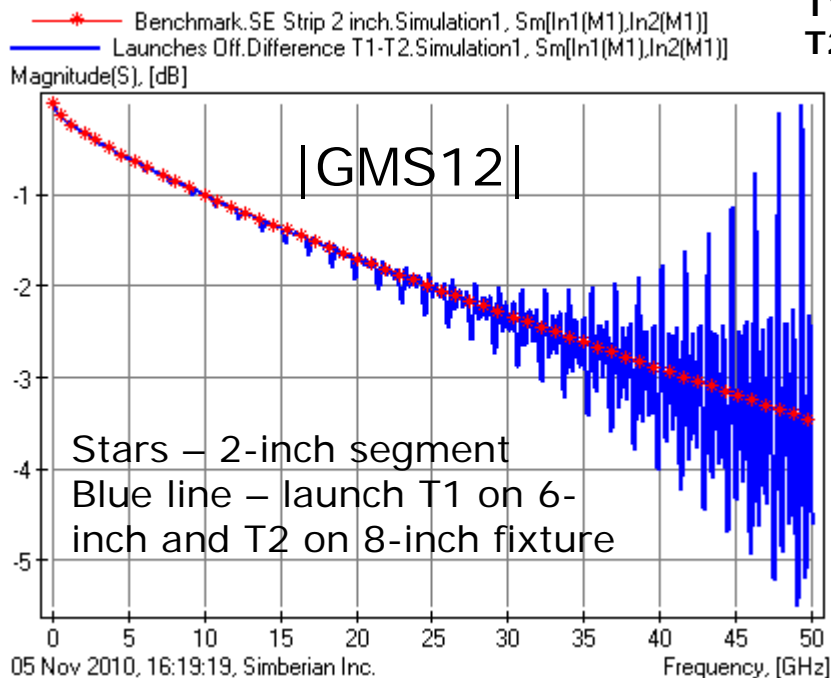
Another pair of launches

- Generalized Modal transmission looks “noisy”
- Material identification may be possible only up to 20-25 GHz

Pad diameter:

T1 – 11.5 mil

T2 – 15 mil

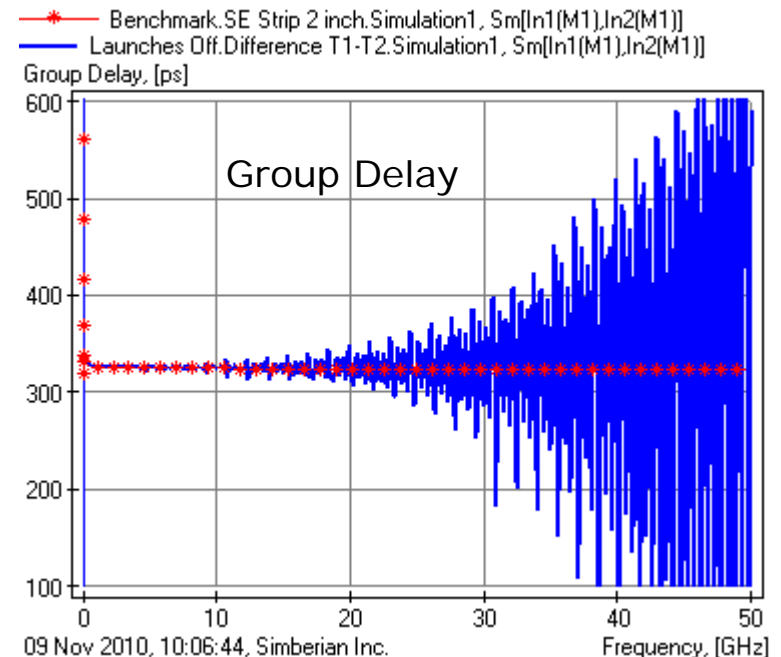
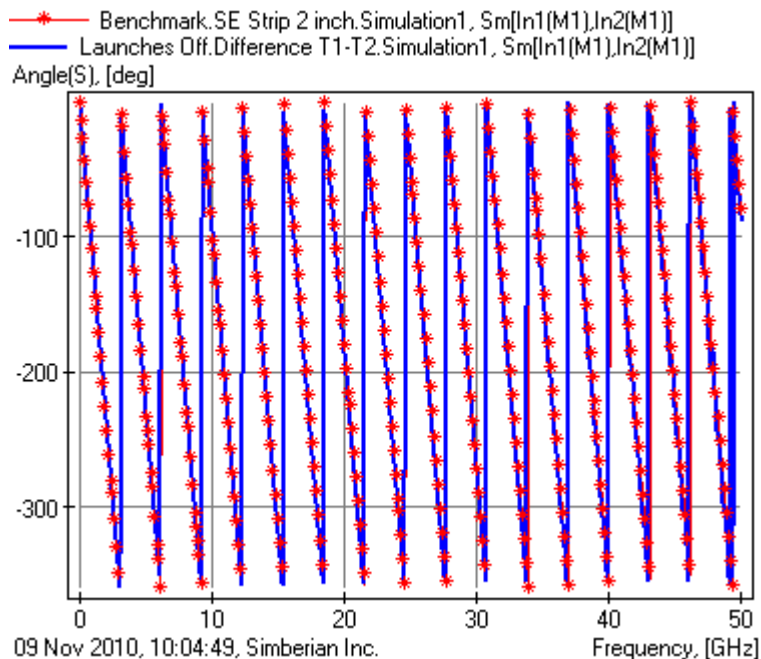


The smaller difference in the reflection loss, the smaller the noise!

Another pair of launches

- Phase of Generalized Modal transmission looks OK
- Group Delay may be usable up to 20 GHz

Arg(GMS12)



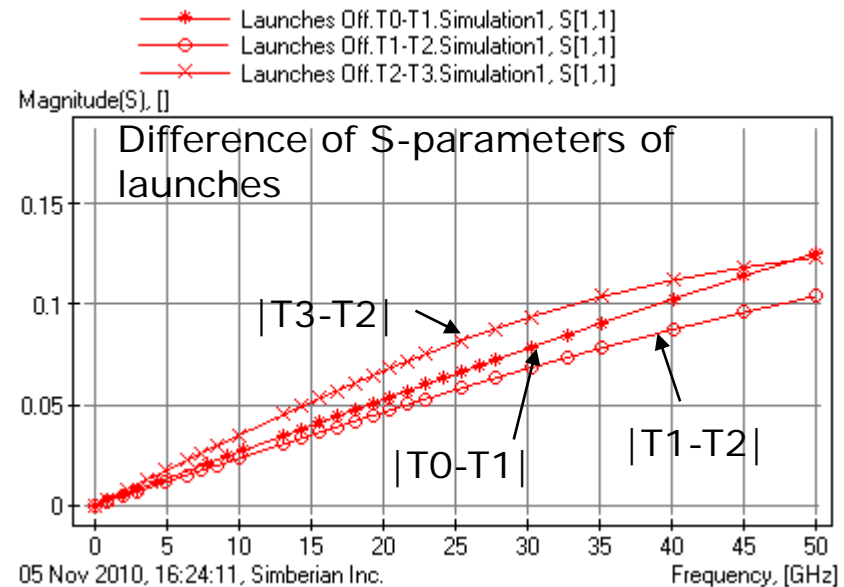
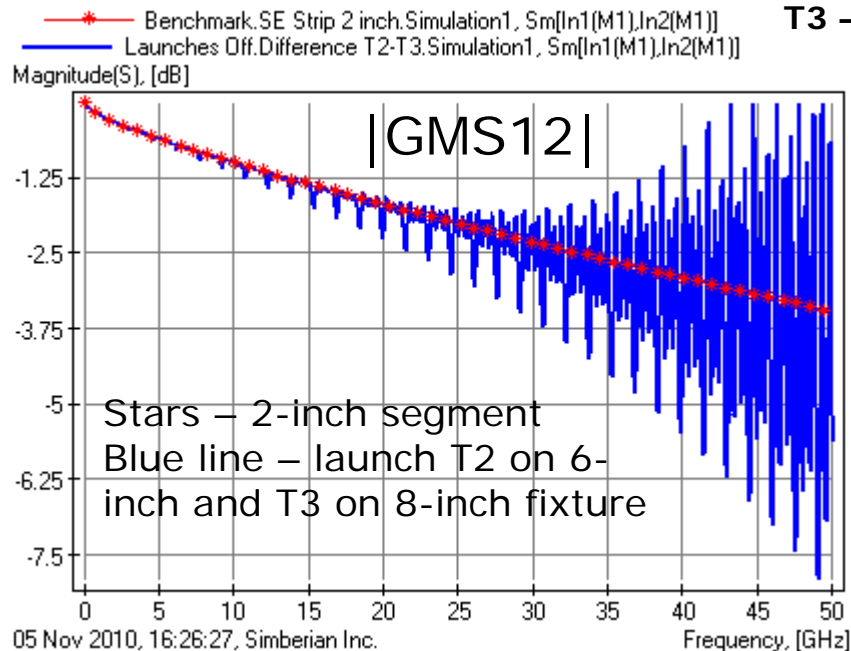
Stars – 2-inch segment

Blue line – launch T1 on 6-inch and T2 on 8-inch fixture

Another pair of launches

- Generalized Modal transmission looks “noisy”
- Material identification may be possible only up to about 15-20 GHz

Pad diameter:
 T2 – 15 mil
 T3 – 18.5 mil

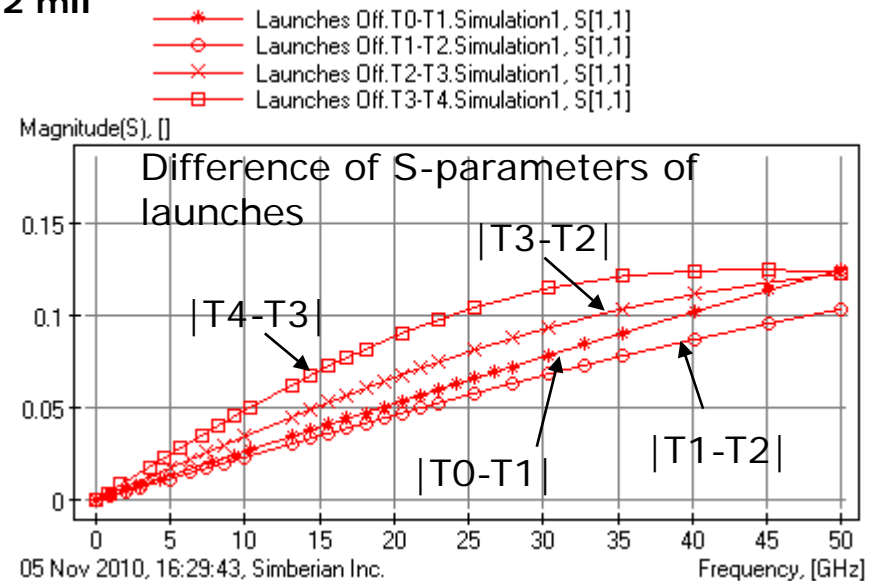
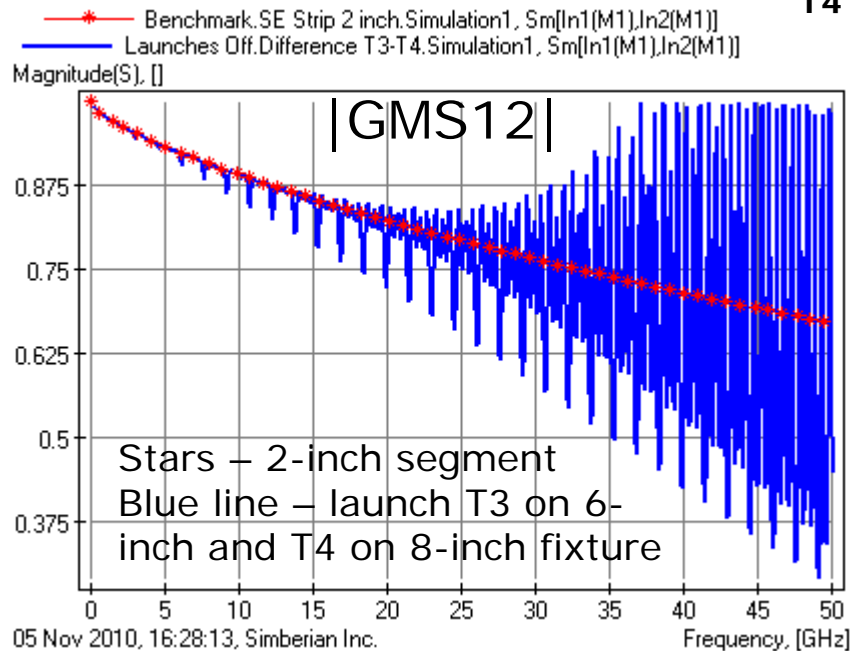


The larger the difference in the reflection loss, the larger the noise!

Worst pair of launches (most reflective)

- Generalized Modal transmission looks “noisy”
- Material identification may be possible only up to about 10-15 GHz

Pad diameter:
T3 – 18.5 mil
T4 – 22 mil

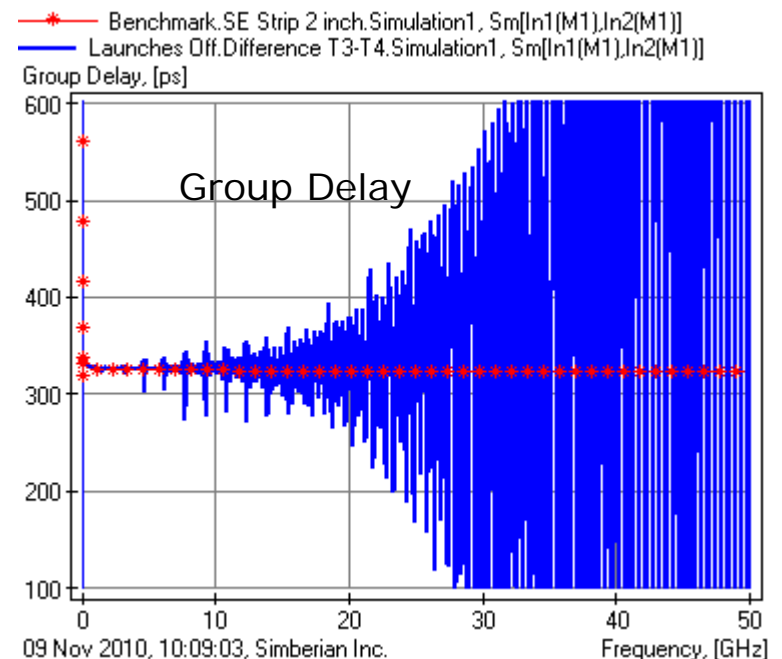
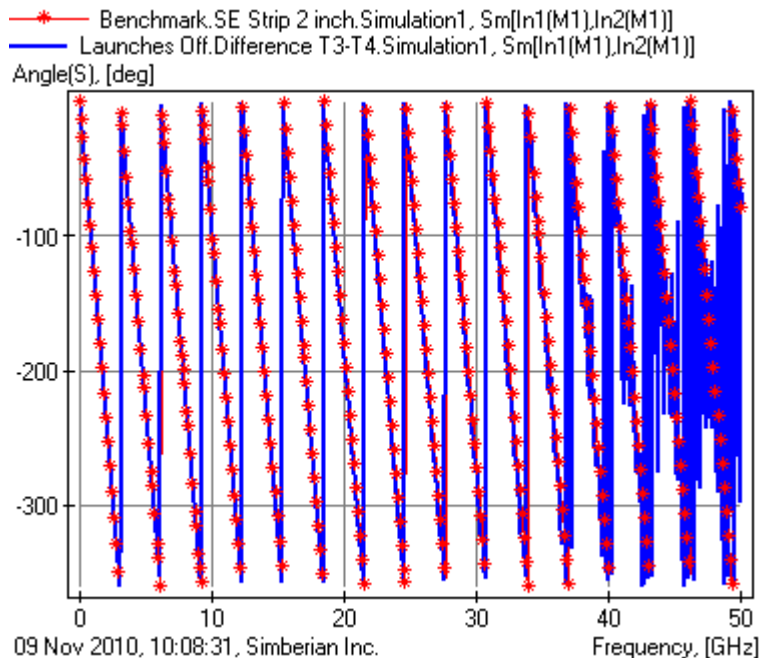


The larger the difference in the reflection loss, the larger the noise!

Worst pair of launches (most reflective)

- Phase becomes “noisy” above 30 GHz
- Group delay is usable only up to 5-10 GHz

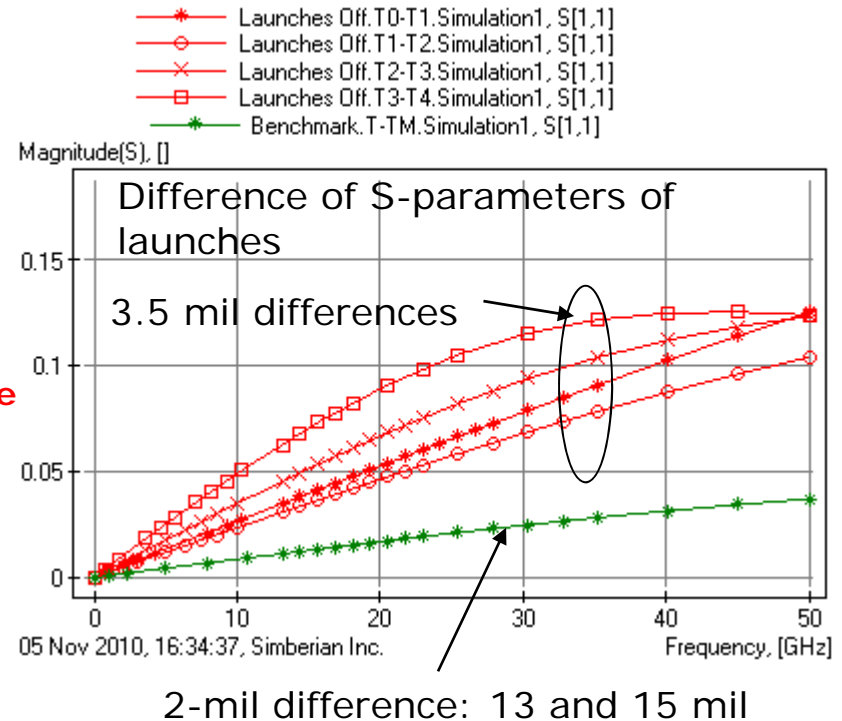
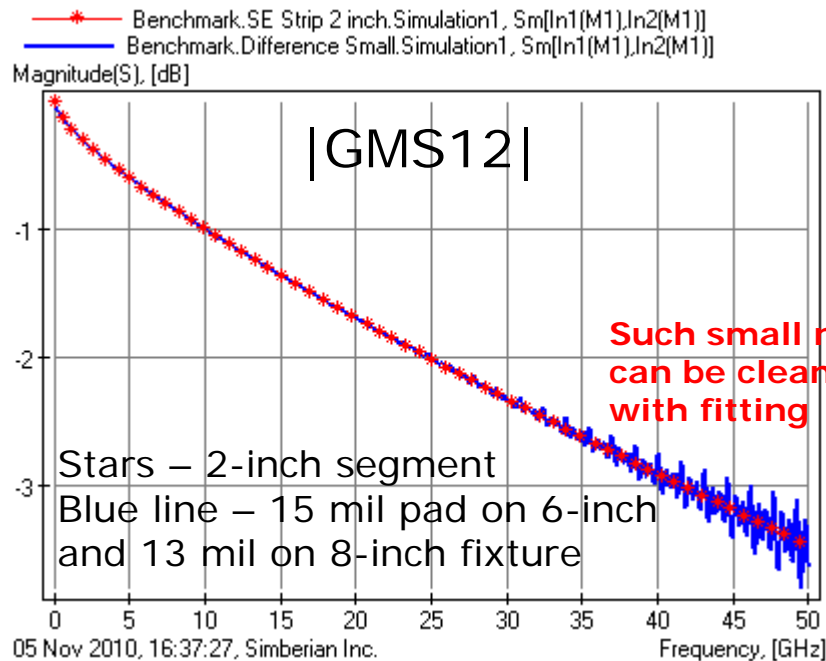
Arg(GMS12)



Stars – 2-inch segment

Blue line – launch T3 on 6-inch and T4 on 8-inch fixture

Example with acceptable difference in pad diameters

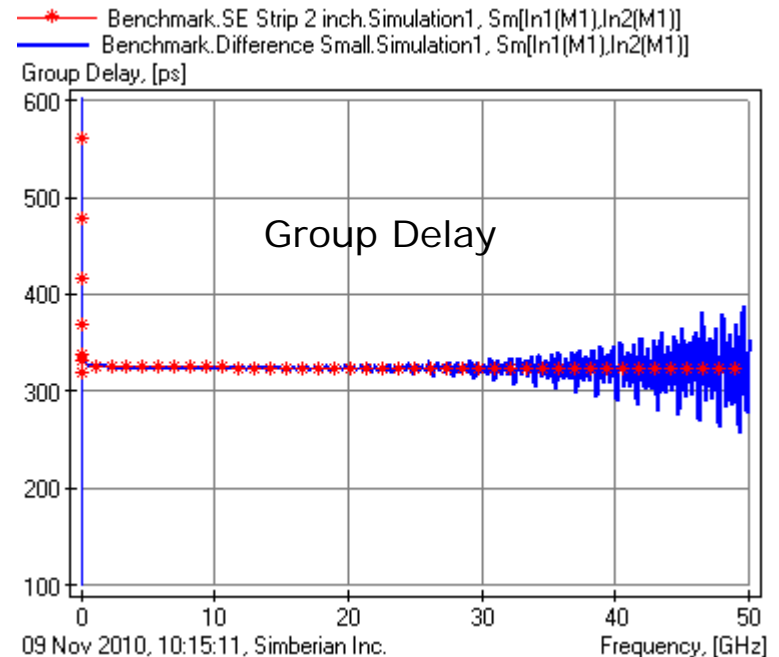
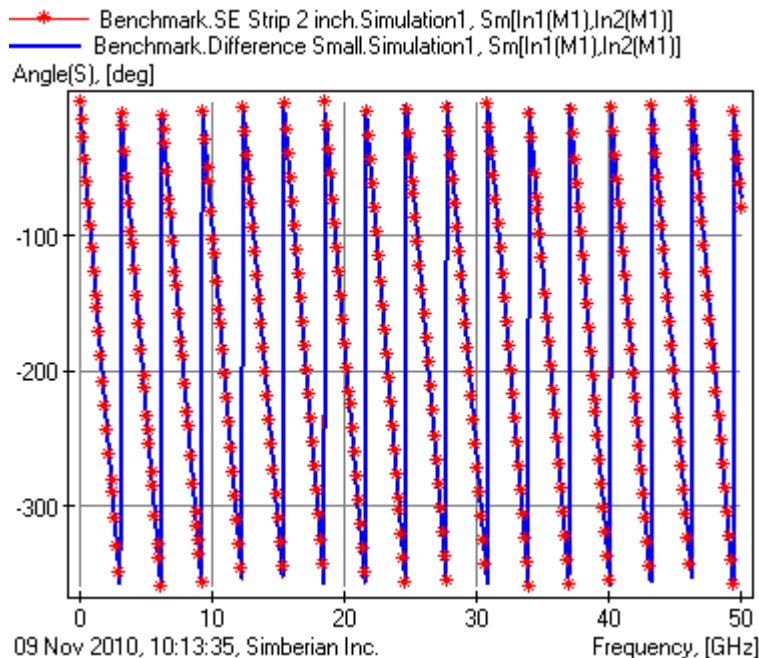


Difference of reflections from the launches should be less than 0.05 for material identification up to 50 GHz
 This measure is not practical – TDR may be used instead

Example with acceptable difference in pad diameters

- Phase is clean and can be used for identification up to 50 GHz
- Group Delay is usable up to 30-40 GHz

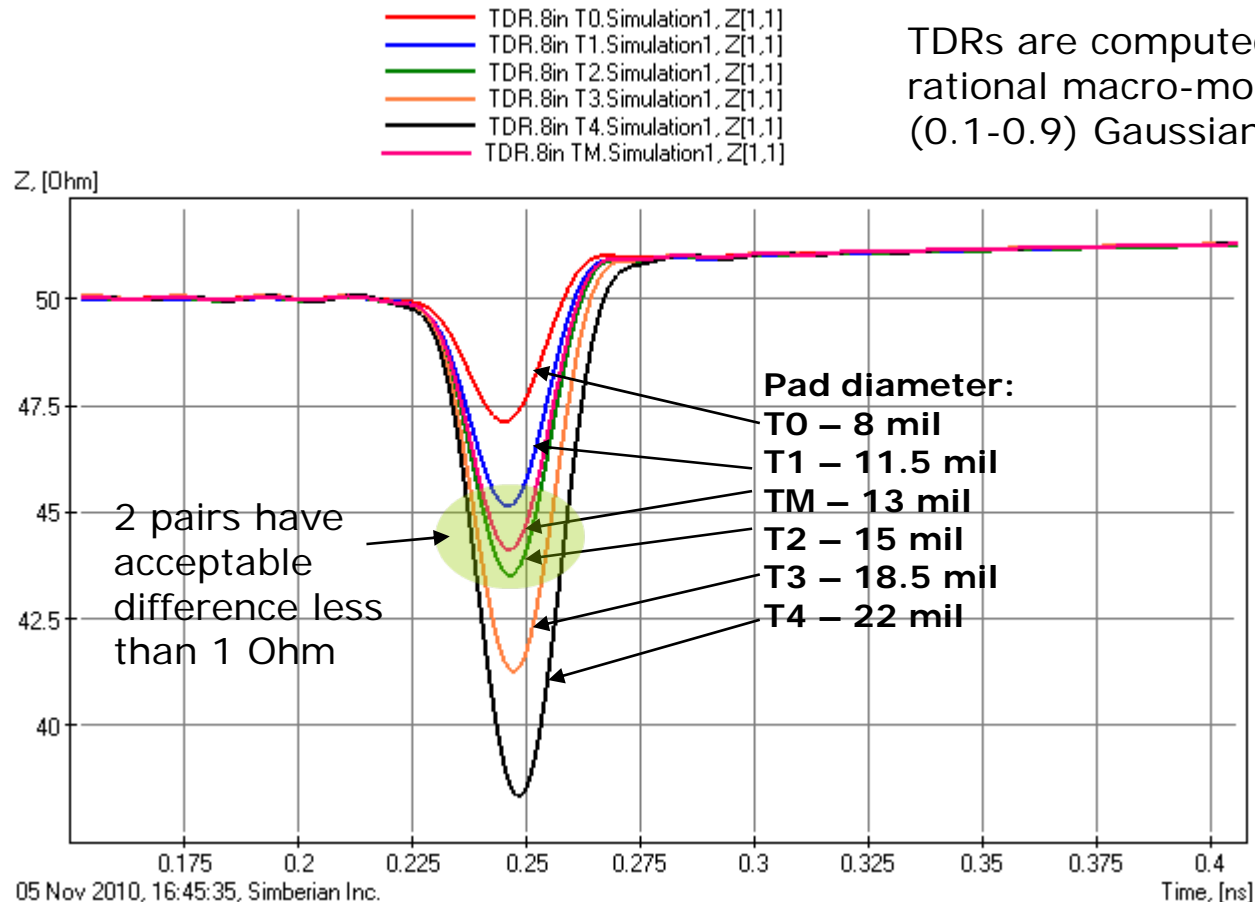
Arg(GMS12)



Stars – 2-inch segment

Blue line – 15 mil pad on 6-inch and 13 mil on 8-inch fixture

TDR of the test fixture can provide measure of non-identity



The difference in the launch impedances should be less than 1 Ohm for material identification up to 50 GHz

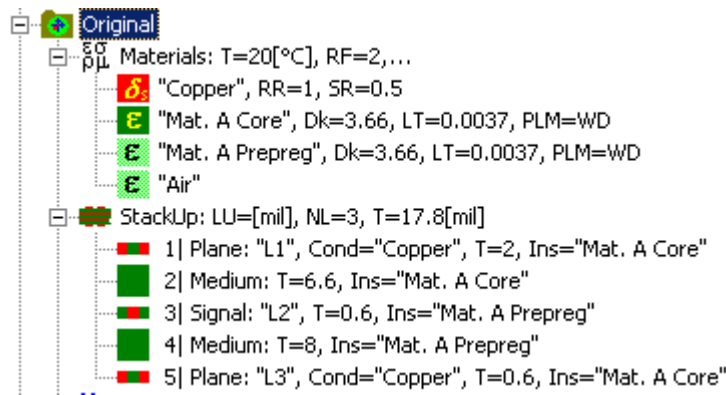
Overview

- Introduction
- Identification of material parameters with Generalized Modal S-parameters
- Sensitivity of GMS-parameters to non-identity of cross-sections
- Sensitivity of GMS-parameters to non-identity of launches
- **Effect of bends on GMS-parameters**
- Conclusion

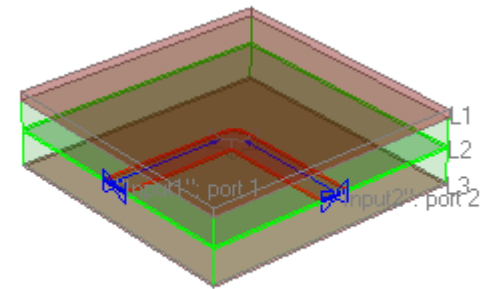
What if line segments in test fixtures are not straight?

□ Numerical experiment to investigate effect of bends

Materials & Stackup

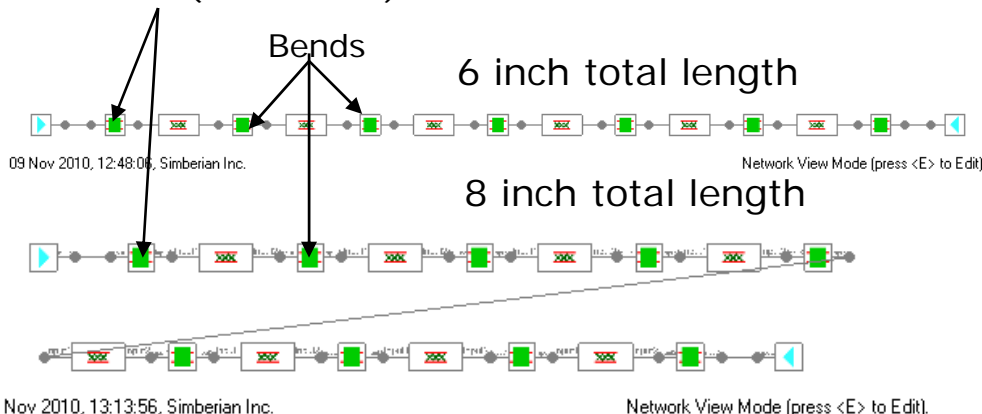


Instead of straight line try to use lines with multiple regular and chamfered bends (from 5 to 15 bends per test fixture)



09 Nov 2010, 12:39:19, Simberian Inc. 3D View Mode (press <E> to Edit).

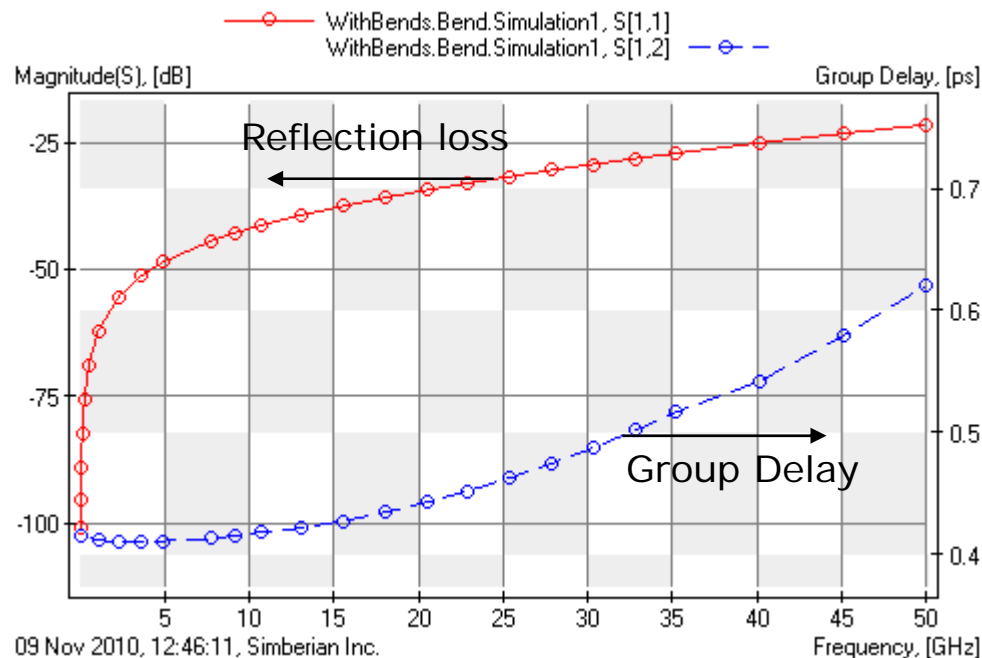
Launches (all identical)



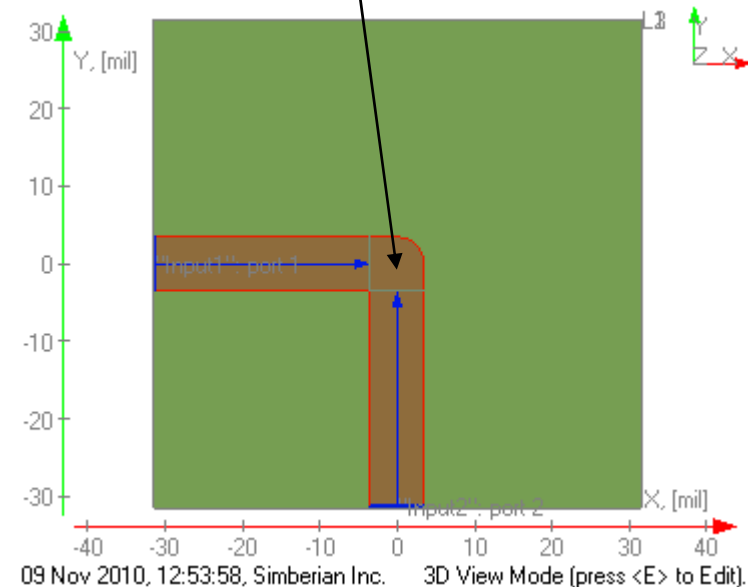
From simulated S-parameters of 2 structures with multiple bends we extract GMS-parameters of 2-inch segment and compare it with the GMS-parameters of 2-inch segment computed directly

S-parameters of the bend

- Very small reflection – below -25 dB up to 40 GHz
- Small additional group delay
- $|S_{12}|$ is very close to 1

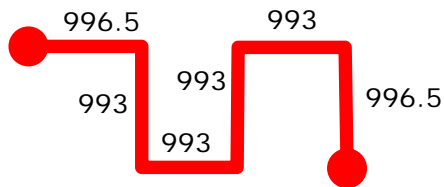


Bend in 7-mil strip line (7 mil of additional length along the strip center line)

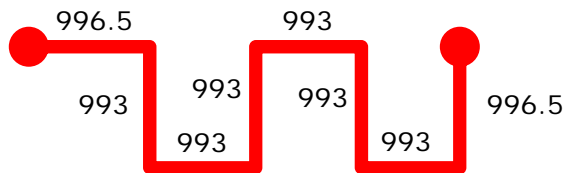


Almost negligible!?

Test fixtures for the extraction



6 inch 5 bends – all line segments 993 mil except first and last one (996.5), with regular or chamfered bends



8 inch 7 bends – all line segments 993 mil except first and last one (996.5), with regular or chamfered bends



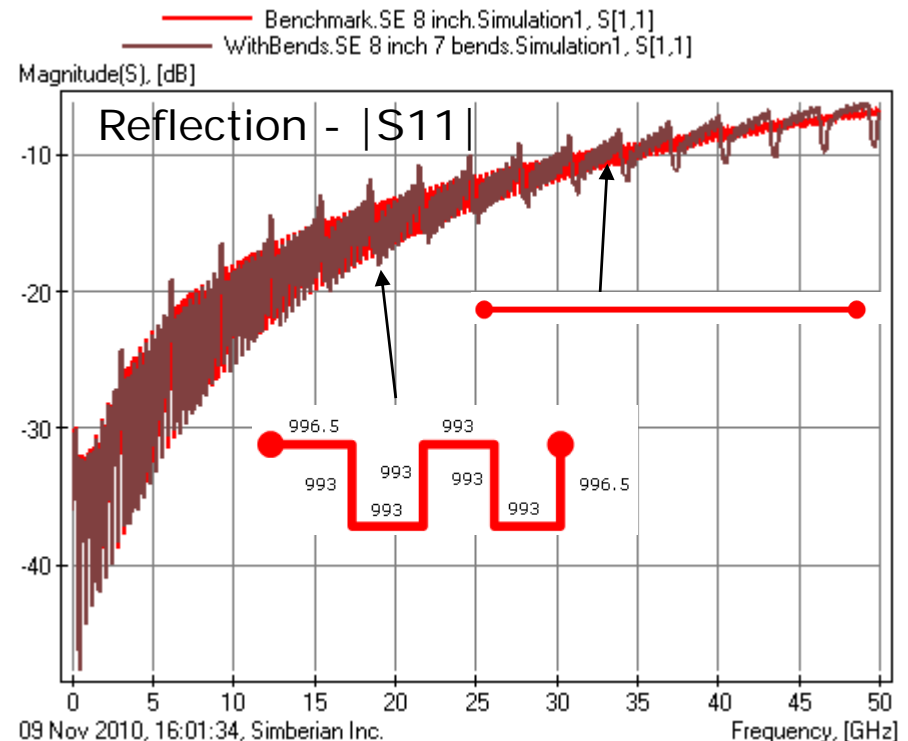
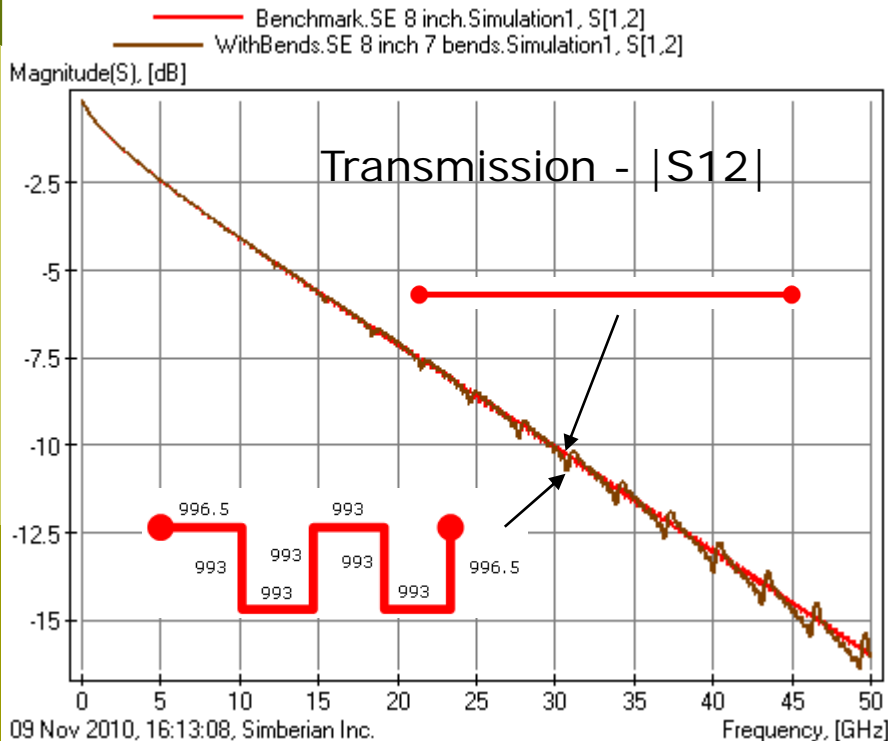
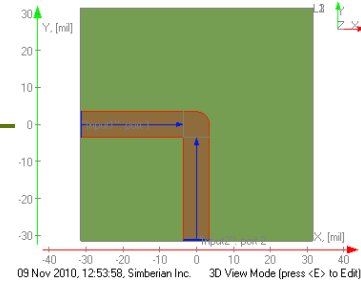
6 inch 11 bends – all line segments 493 mil except first and last one (496.5), with regular or chamfered bends



8 inch 15 bends – all line segments 493 mil except first and last one (496.5), with regular or chamfered bends

Effect of bends on 8-inch test fixture

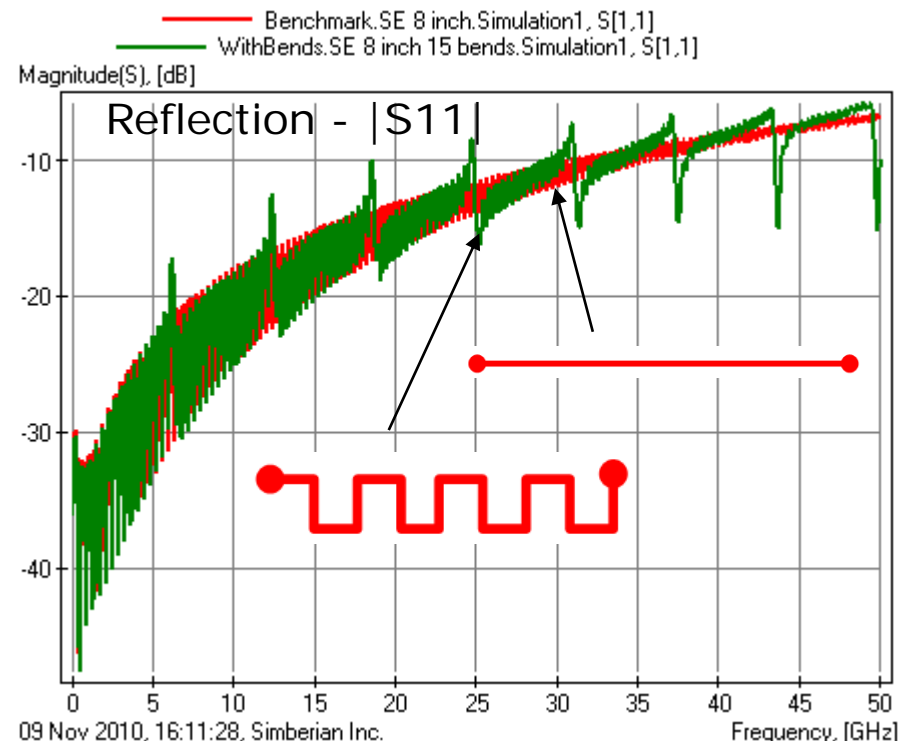
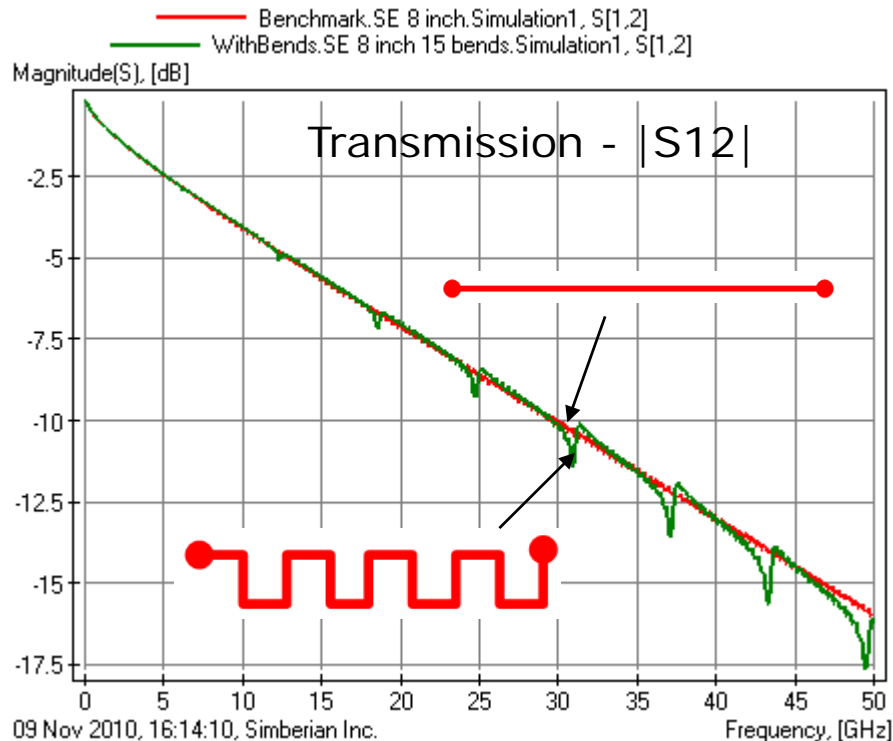
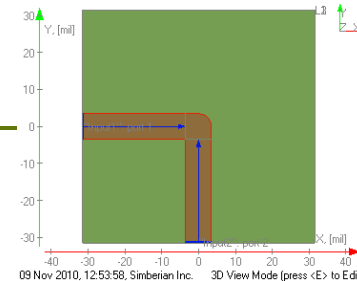
- 7 regular bends cause small resonances in reflection and transmission and decrease group delay
- Looks like nothing to worry up to 20 GHz



Red lines – 8 inch straight;
Brown lines – 8 inch with 7 bends

Effect of bends on 8-inch test fixture

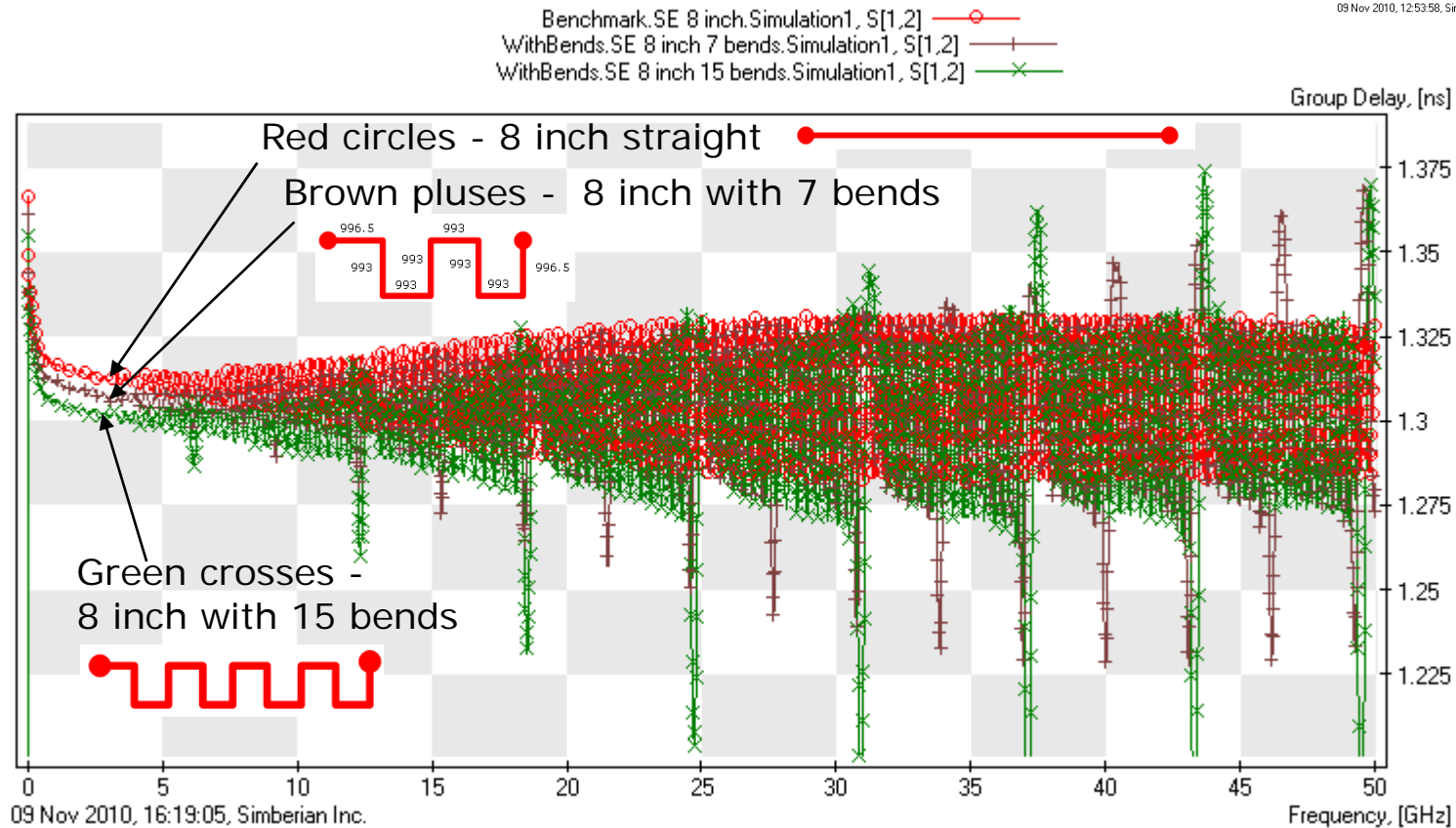
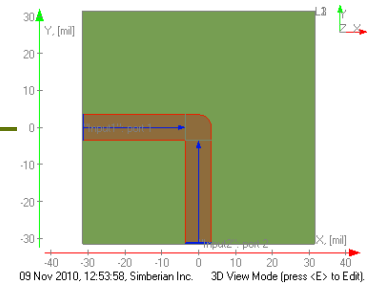
- 15 regular bends cause more visible resonances in reflection and transmission and decrease group delay
- Though, it looks like nothing to worry up to 20 GHz



Red lines – 8 inch straight;
Green lines – 8 inch with 15 bends

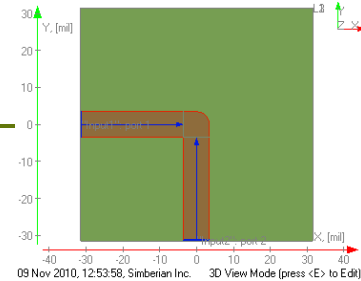
Effect of bends on 8-inch test fixture

- Group delay with bends is smaller and has spikes at resonances

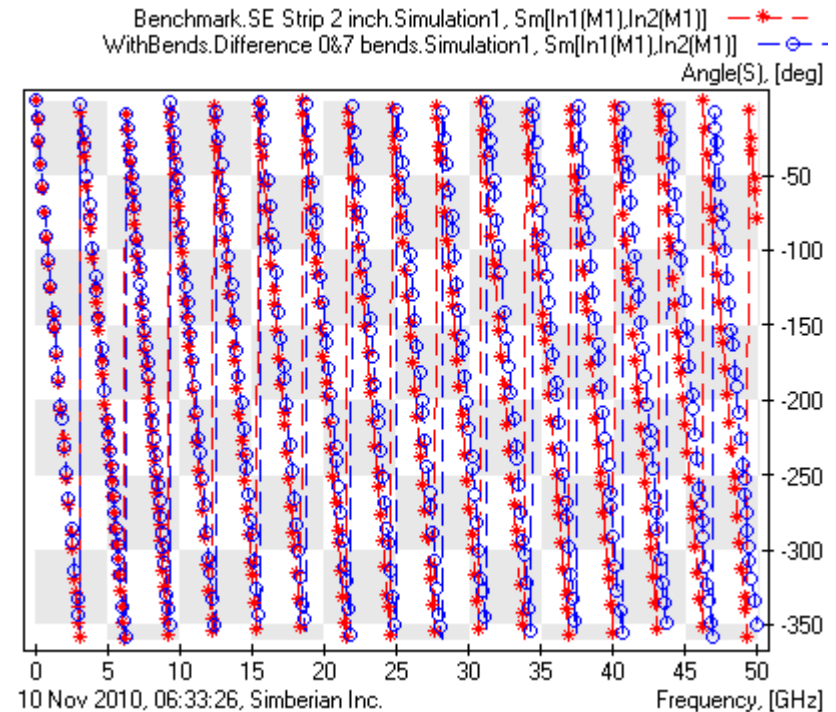
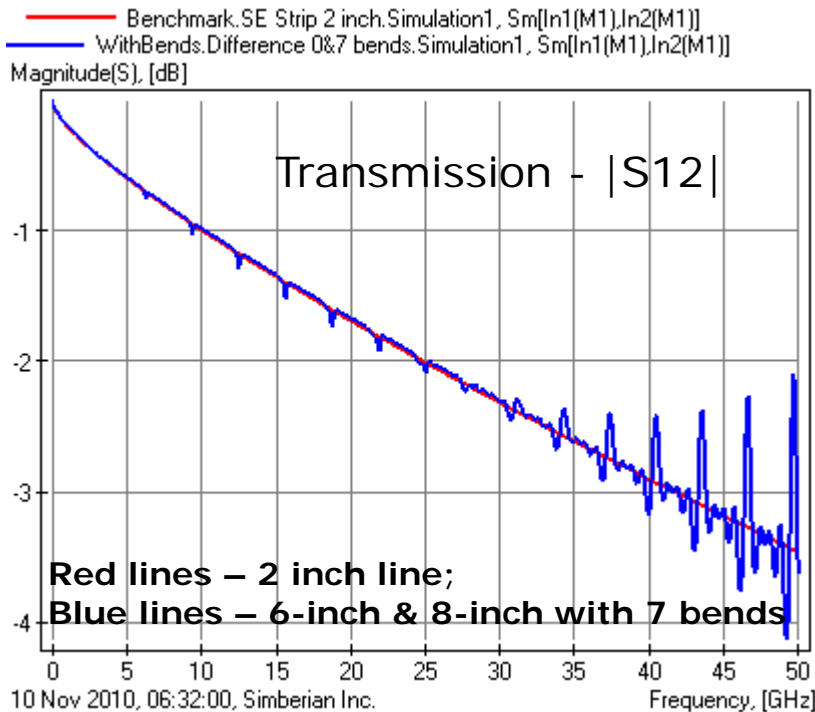


Effect of bends on GSM-parameters

- 6-inch straight line + 8-inch line with 7 bends



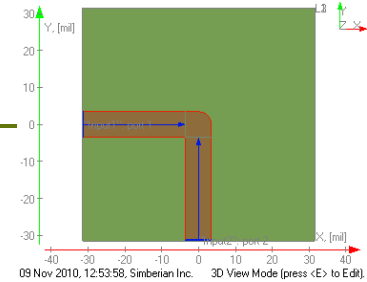
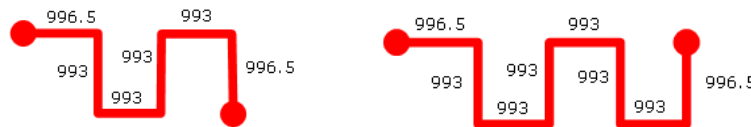
Angle of |S12|



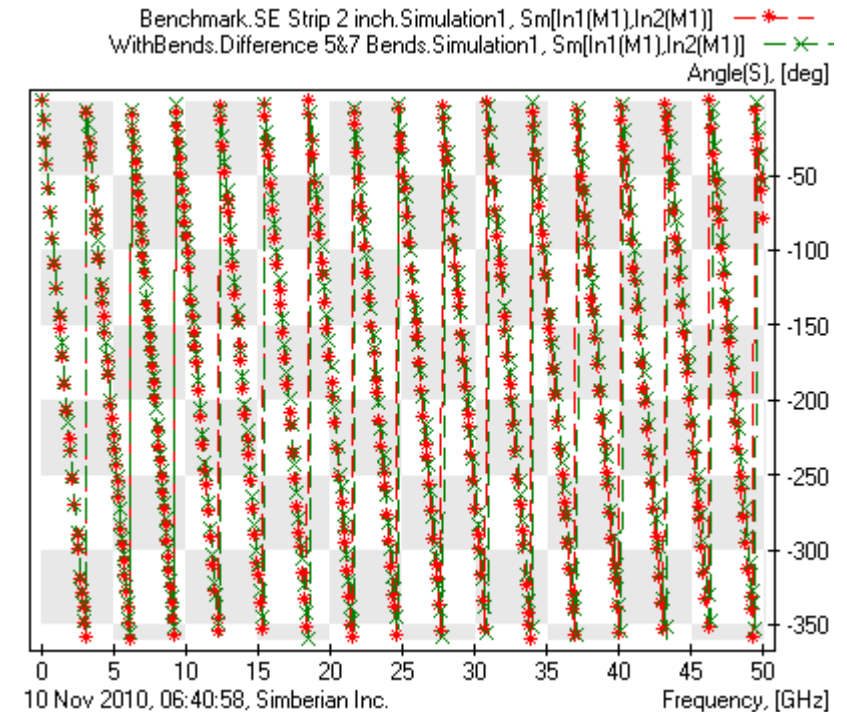
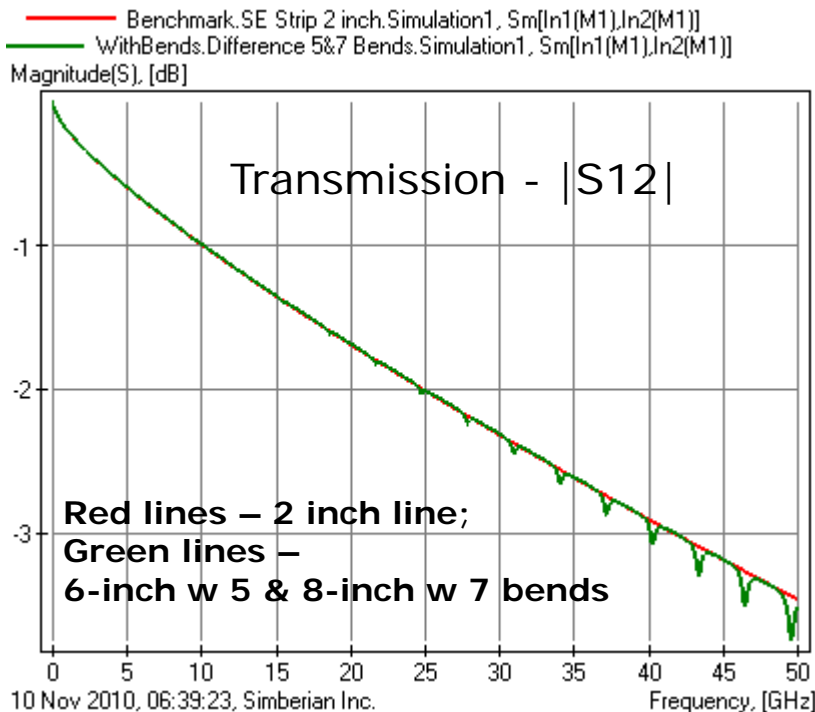
Noise above 30 GHz and under-estimated angle and DK!

Effect of bends on GMS-parameters

- 6-inch with 5 bends + 8-inch with 7 bends



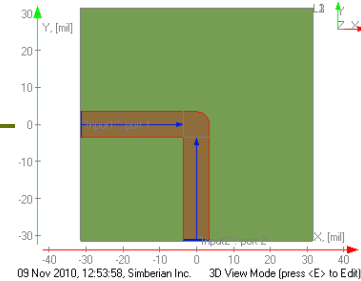
Angle of S12



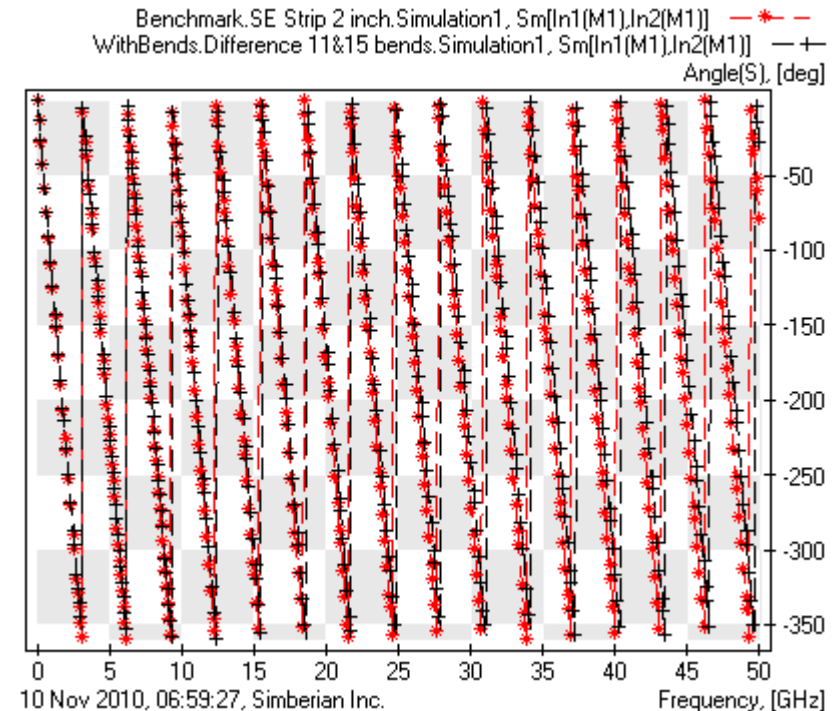
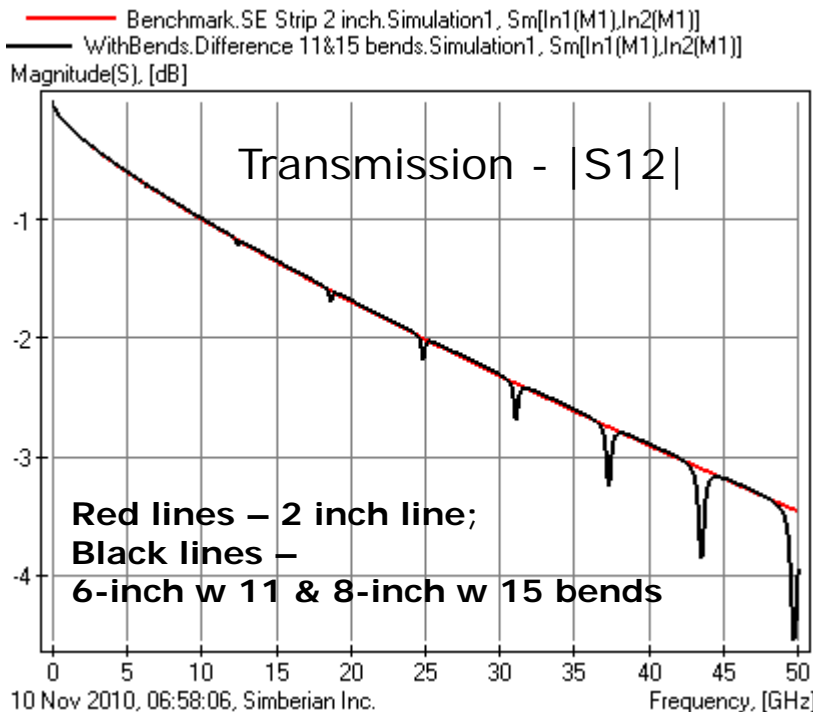
Small dips above 30 GHz and slightly under-estimated phase
 Matching number of bends may be good idea!

Effect of bends on GMS-parameters

- 6-inch with 11 bends + 8-inch with 15 bends



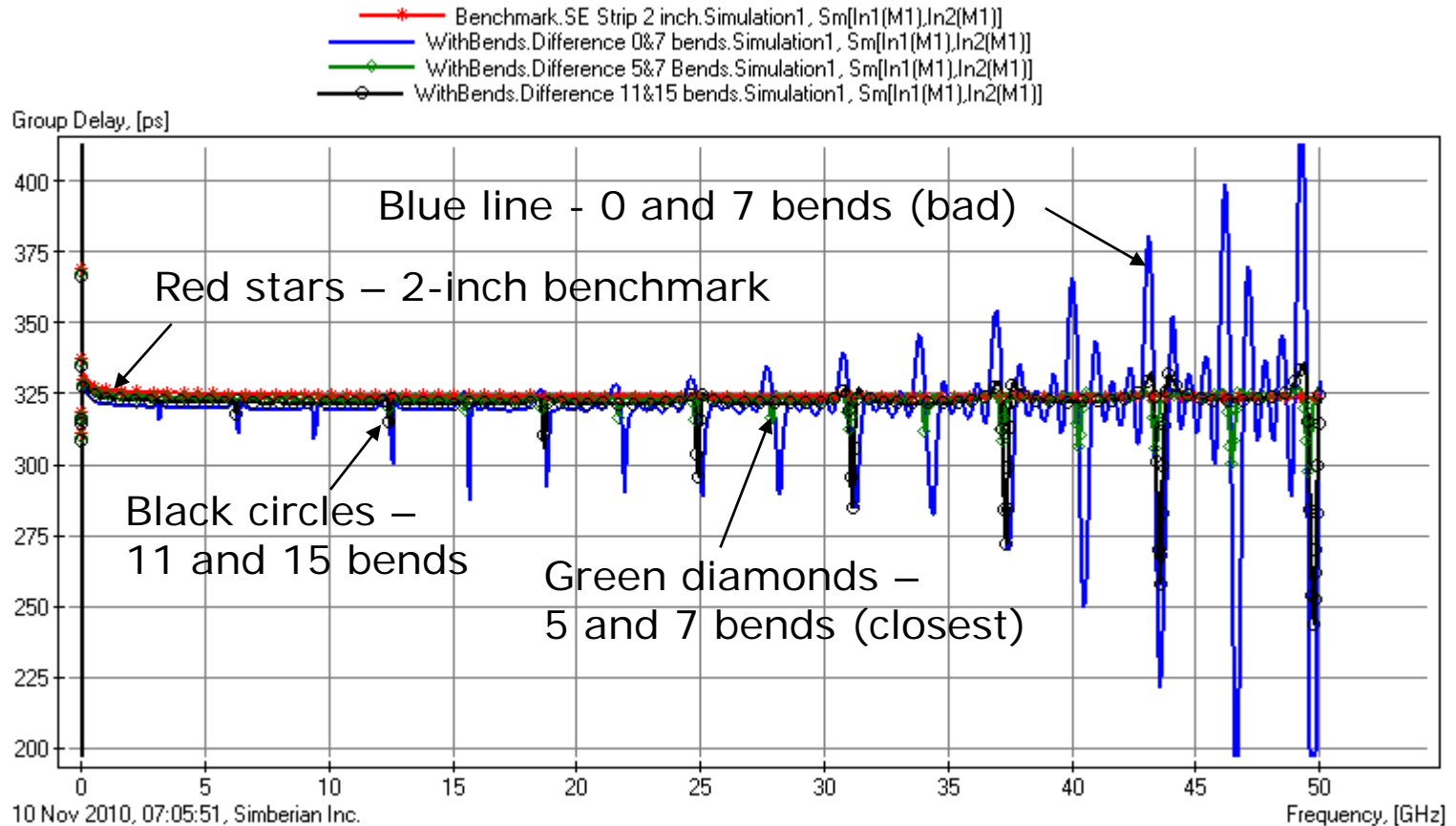
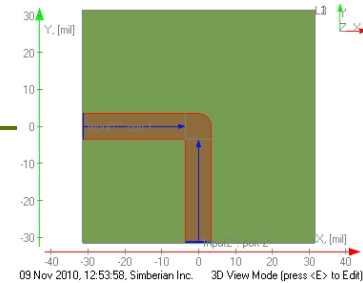
Angle of S12



Larger dips above 25 GHz and under-estimated phase (and DK)
 Magnitude of dips is sensitive to total number of bends and phase is sensitive to the difference in the bend count!

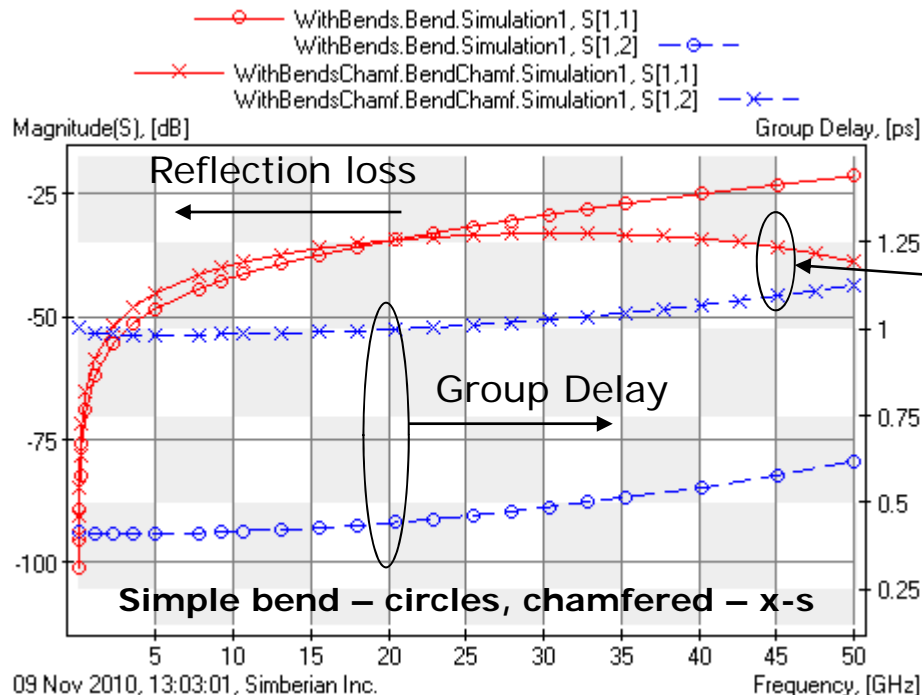
Effect of bends on GMS-parameters

- Group delay is smaller and more noisy for larger difference in the bend count

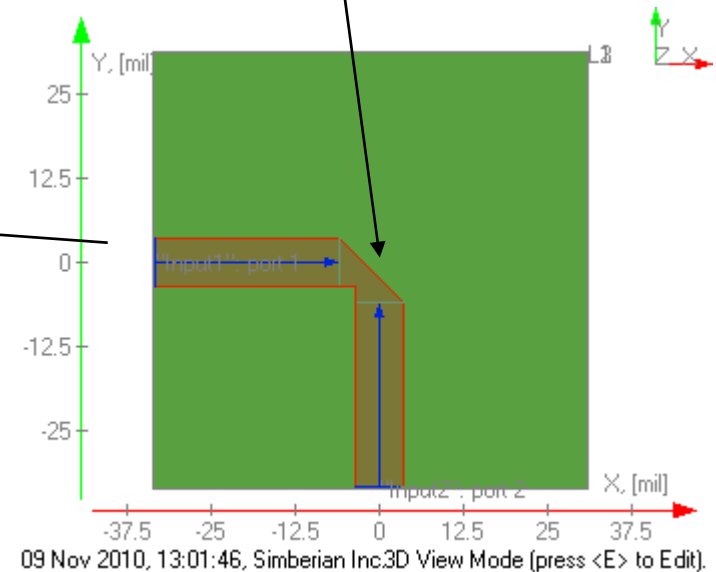


What if we compensate bends with chamfers?

- Extremely small reflection – below -30 dB up to 50 GHz!
- Small additional group delay
- $|S_{12}|$ is very close to 1

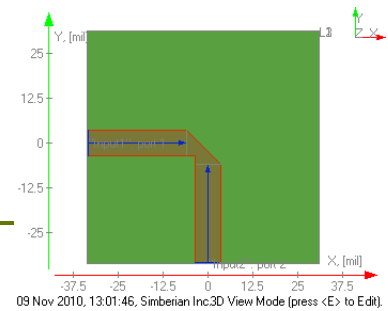


Chamfered bend in 7-mil strip line (12 mil of additional length along the strip center line)



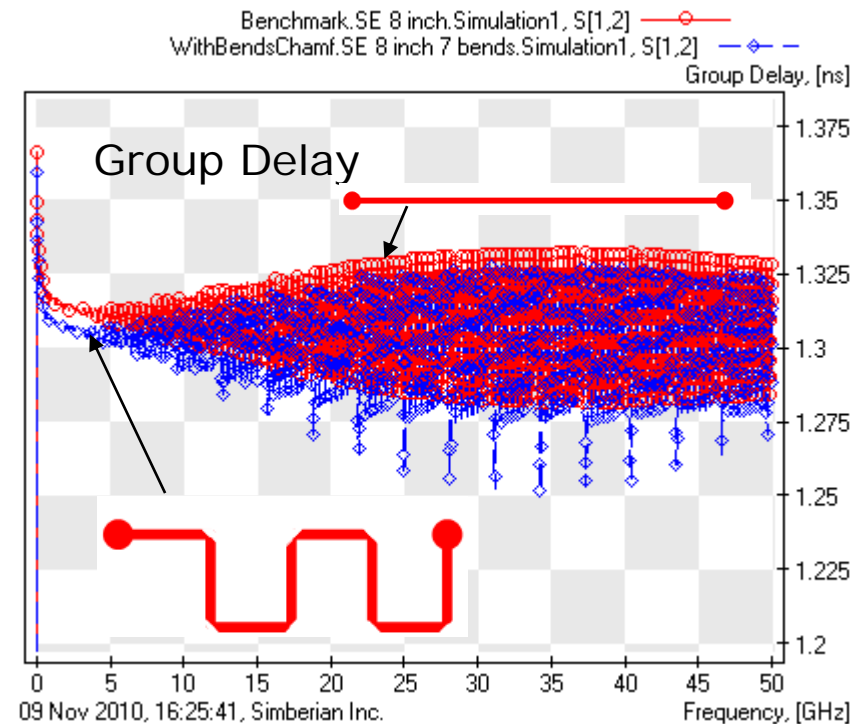
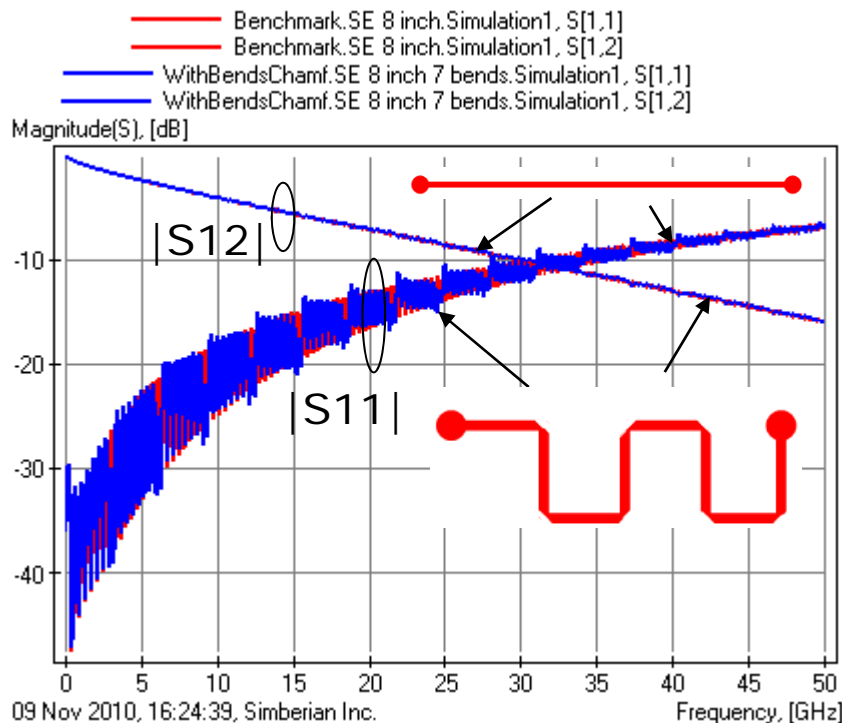
Negligible for sure?!

Effect of compensation on 8-inch test fixture



- Nearly perfect match in magnitudes
- Group delay is smaller – difficult to guess actual signal path

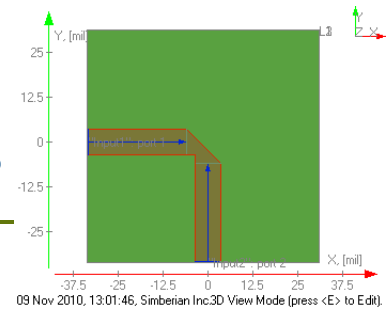
Length of meandering line is measured along the center line



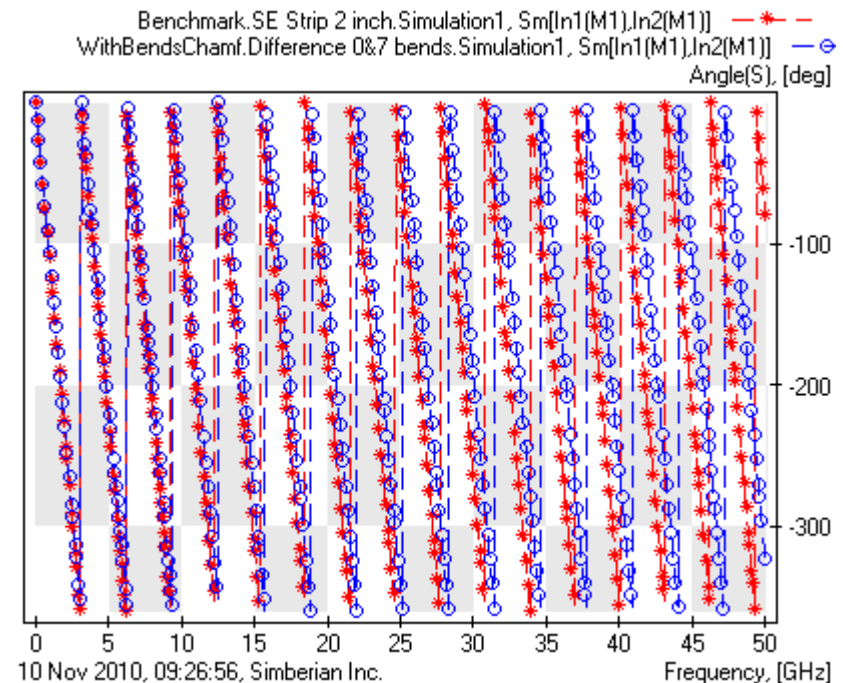
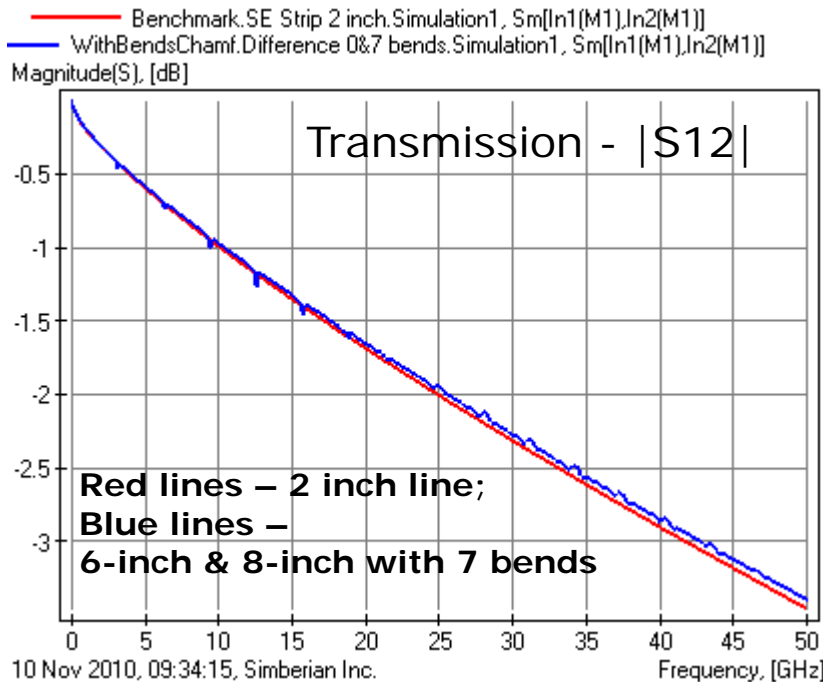
Red lines – 8 inch straight; Blue lines – 8 inch with 7 chamfered bends

Effect of bends on GMS-parameters

- 6-inch straight line + 8-inch line with 7 chamfered bends



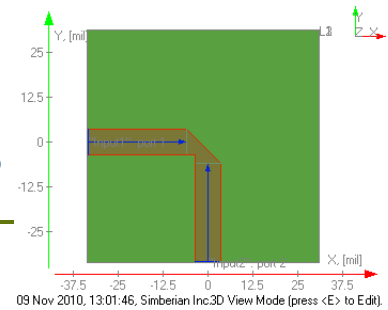
Angle of $|S_{12}|$



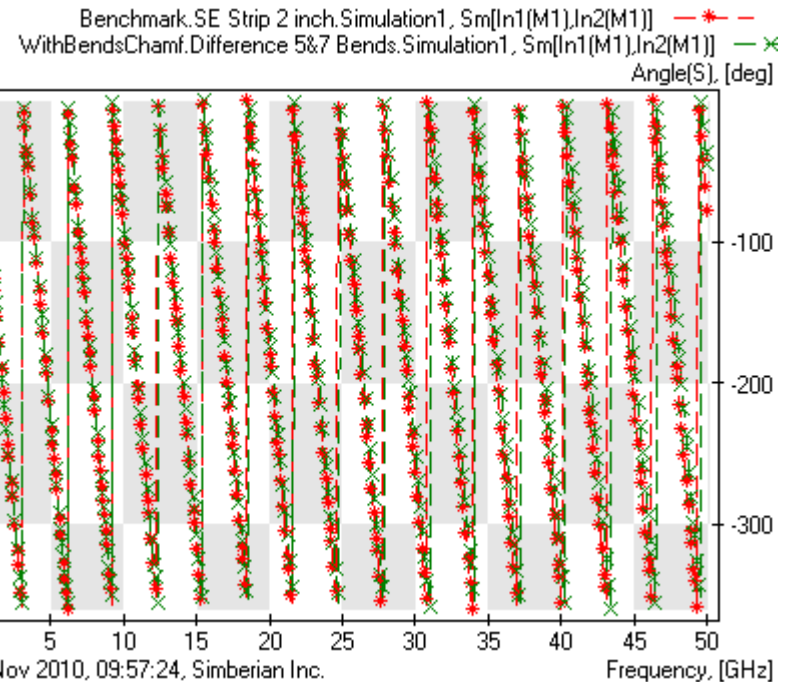
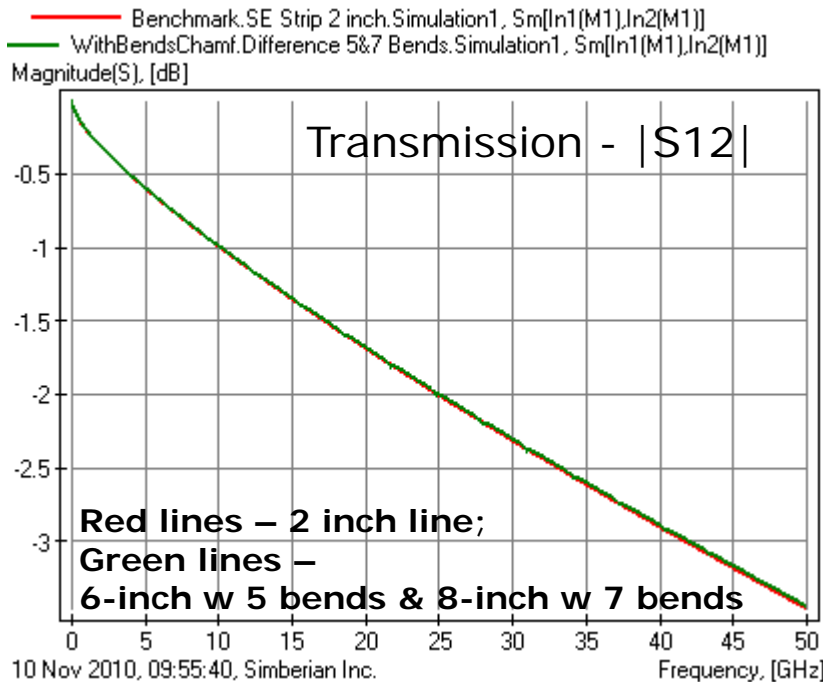
Slightly smaller attenuation (lower LT) and under-estimated angle (lower DK)!

Effect of bends on GMS-parameters

- 6-inch with 5 chamf. bends + 8-inch with 7 chamf. bends



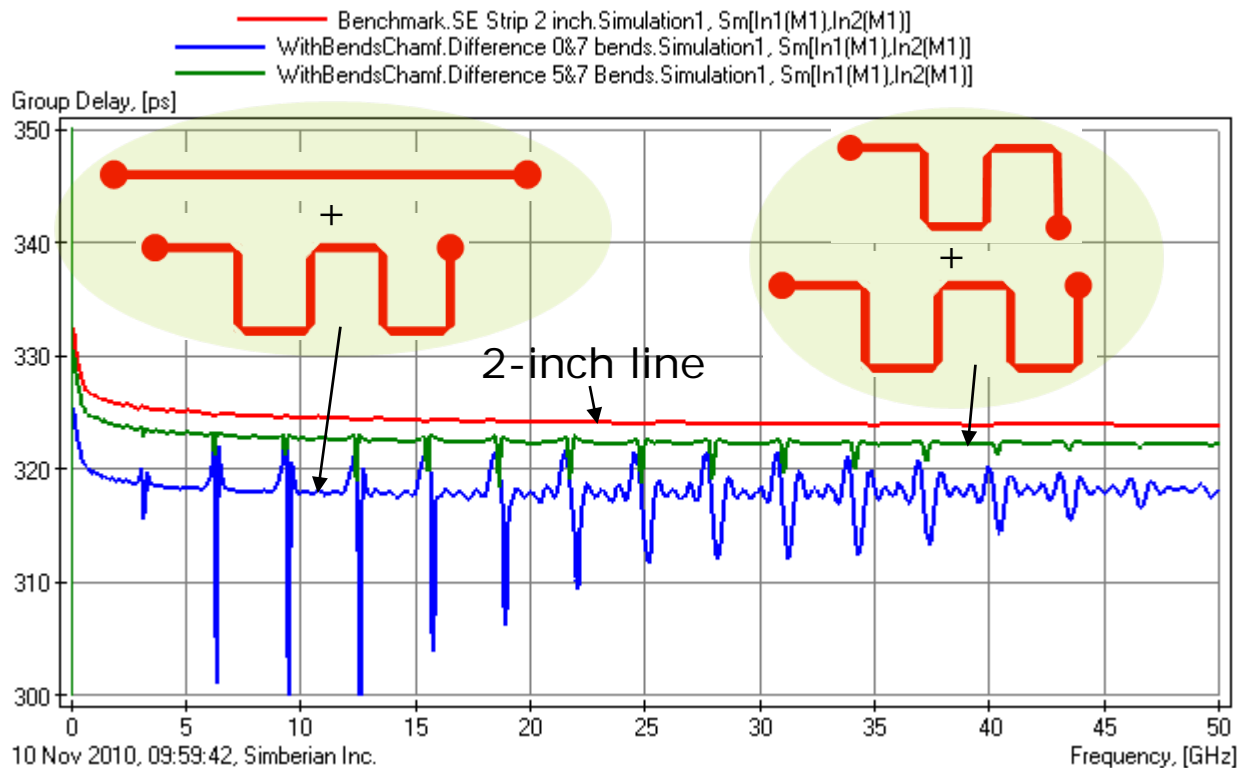
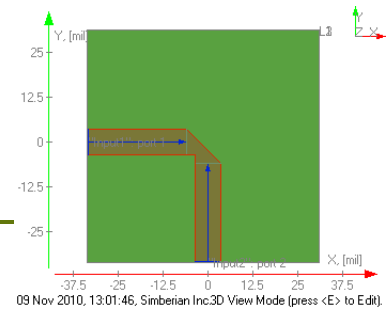
Angle of $|S_{12}|$



Ideal attenuation and slightly under-estimated angle (lower DK)

Effect of chamfered bends on group delay

- About 6 ps difference in case of no bends in one test fixture (may lead to about 3% difference in identified DK)
- Less than 2 ps difference in case of 5 and 7 chamfered bends – acceptable for the extraction of DK (about 1% difference in identified DK)



Conclusion

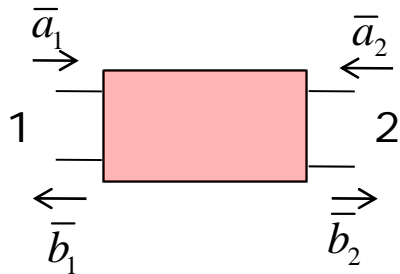
- Overview of material parameters identification by fitting measured and computed GMS-parameters for single strip line is provided
- Sensitivity of the GMS-parameters method to variations in geometry of the test fixtures is investigated with numerical experiments
 - Small variations in strip width and shape in the test fixtures can cause over or under-estimation of extracted material loss parameters (LT, conductor and roughness parameters)
 - Small variations in geometry of connectors or launches in test fixtures can cause “noise” in GMS-parameters at high frequencies – TDR may be used to qualify test fixtures
 - Multiple bends or other discontinuities can cause noise in GMS-parameters and under-estimation of dielectric constant
 - Matching number of bends and chamfering reduces the extraction errors
- Setting up all simulations and model building with Simbeor took about 3 hours

Solutions and contact

- Simbeor solution files are in the database
<http://kb.simberian.com/SimbeorExamples.php> (keyword 2010_03)
It contains all electromagnetic models and linear circuit analysis both in frequency and time domains
- Send questions and comments to
 - General: info@simberian.com
 - Sales: sales@simberian.com
 - Support: support@simberian.com
- Web site www.simberian.com

S-matrices and T-matrices

Same number of ports on the left and right side of multiport



$$\begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \end{bmatrix} = \begin{bmatrix} S_{1,1} & S_{1,2} \\ S_{2,1} & S_{2,2} \end{bmatrix} \cdot \begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \end{bmatrix}$$

Cascading of 2 multiports described with S-parameters require solving a linear system

$$\begin{bmatrix} \bar{b}_1 \\ \bar{a}_1 \end{bmatrix} = \begin{bmatrix} T_{1,1} & T_{1,2} \\ T_{2,1} & T_{2,2} \end{bmatrix} \cdot \begin{bmatrix} \bar{a}_2 \\ \bar{b}_2 \end{bmatrix}$$

Cascading of 2 multiports described with T-parameters is simple product of two T-matrices

$$\begin{aligned} T_{1,1} &= S_{2,1} - S_{1,1} \cdot S_{2,1}^{-1} \cdot S_{2,2} \\ T_{1,2} &= S_{1,1} \cdot S_{2,1}^{-1} \\ T_{2,1} &= -S_{2,1}^{-1} \cdot S_{2,2} \\ T_{2,2} &= S_{2,1}^{-1} \end{aligned}$$

$$\begin{aligned} S_{1,1} &= T_{1,2} \cdot T_{2,2}^{-1} \\ S_{1,2} &= T_{1,1} - T_{1,2} \cdot T_{2,2}^{-1} \cdot T_{2,1} \\ S_{2,1} &= T_{2,2}^{-1} \\ S_{2,2} &= -T_{2,2}^{-1} \cdot T_{2,1} \end{aligned}$$

All elements are scalars in case of 2-ports (single-ended lines) or matrices in case of multi-conductor lines (differential)

See more in Carlin, Giordano, Network Theory, An Introduction to Reciprocal and Non-Reciprocal Circuits, 1964

Conversion can be generalized for arbitrary number of ports on the left and right