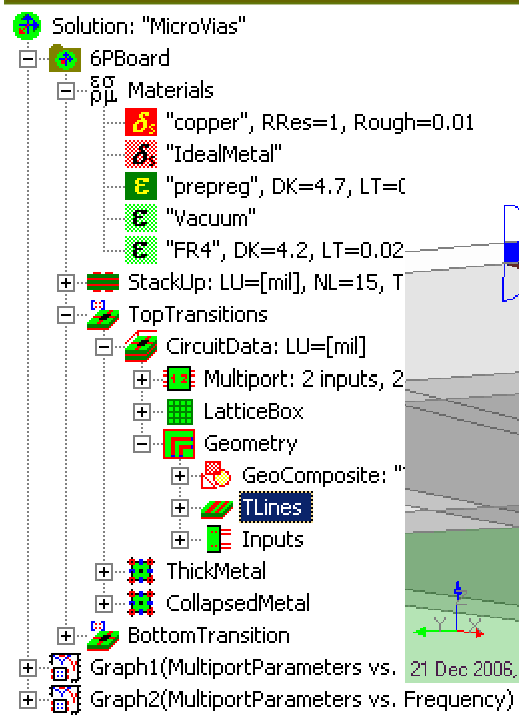
Quality metrics for S-parameter models

DesignCon IBIS Summit, Santa Clara, February 4, 2010



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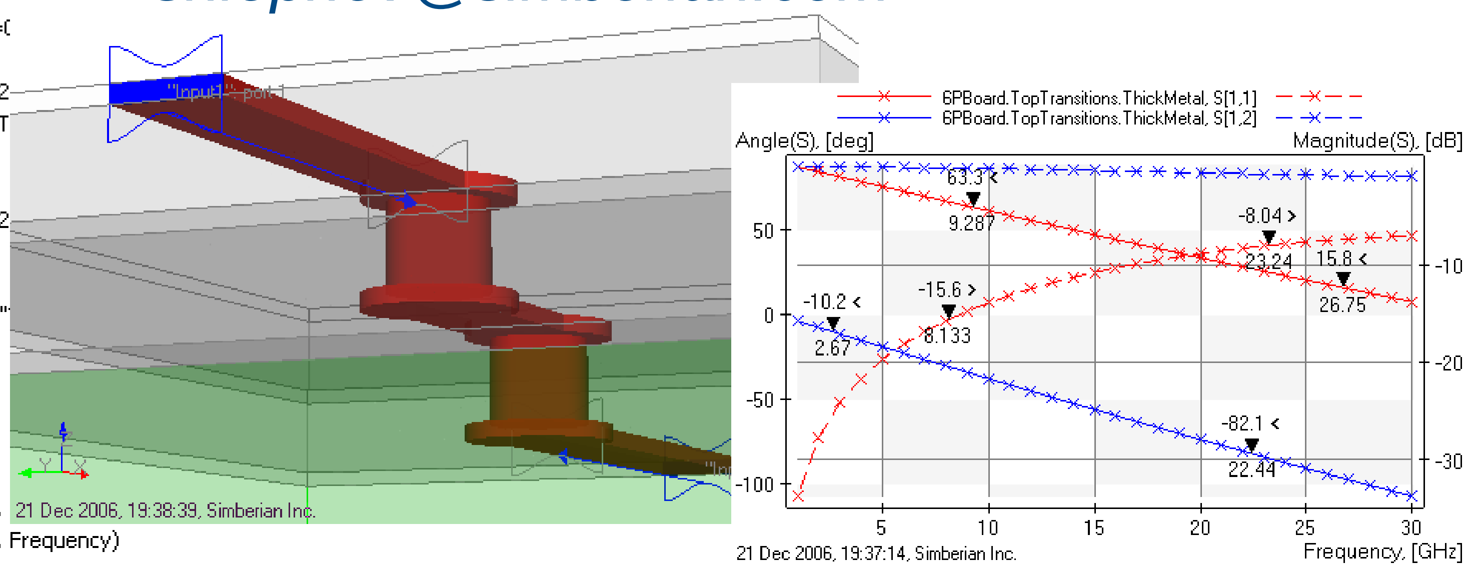


Agenda

- Introduction
- Reciprocity metric
- Passivity metric
- Causality metrics
- Global quality metrics
- Examples
- Conclusion
- Contacts and resources

Introduction

- S-parameter models are becoming ubiquitous in design of multi-gigabit interconnects
  - Connectors, cables, PCBs, packages, backplanes, … can be characterized with S-parameters from DC to daylight
- Such models come from measurement or electromagnetic analysis
- And very often have some quality issues
  - Passivity and reciprocity violations
  - Causality problems

If You happen to…

- Build interconnect models for internal use
- Send interconnect models to customers developing consumer products
- Confirm models with measurements or electromagnetic analysis
- Use models for compliance level testing
- …

You need to have…

Pristine S-parameters

- Reciprocal (no non-linear anisotropic materials)

$$S_{i,j} = S_{j,i} \quad or \;\; S = S^t$$

- Passive (interconnects do not generate energy)

$$P_{in} = \bar{a}^* \cdot \left[U - S^* S \right] \cdot \bar{a} \; \geq 0 \quad \Rightarrow \quad eigenvals\left[S^* \cdot S \right] \leq 1$$

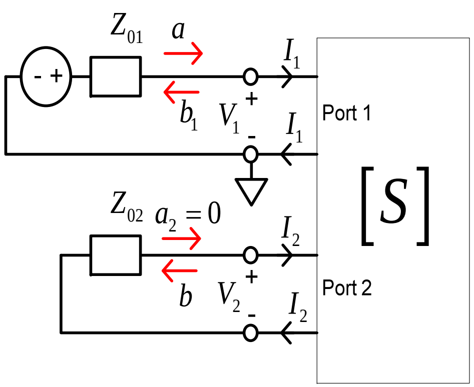
- Causal – no response before the excitation

$$S_{i,j}\left(t\right) = 0, \;\; t < T_{ij}$$

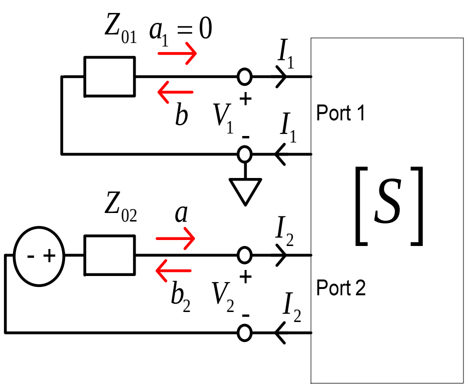
- Stable analysis in time domain
- What if some of those properties are violated – can we still use such model and trust the results?
- This presentation introduces metrics to distinguish good models from bad ones and methodology to improve the model quality for consistent frequency and time-domain analyses

Reciprocity

- Linear circuits with reciprocal materials are reciprocal according to Lorentz's theorem of reciprocity:
  Reflected wave measured at port 2 with incident wave at port 1 is equal to reflected wave measured at port 1 with the same incident wave at port 2



$$b = S_{2,1} \cdot a$$



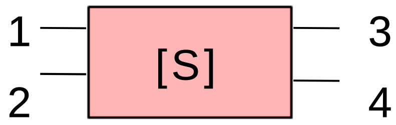
$$b = S_{1,2} \cdot a \quad \Rightarrow \quad S_{2,1} = S_{1,2}$$

- In general it means that the scattering matrices are symmetric

$$S_{i,j} = S_{j,i} \quad or \quad S = S^t$$ at all frequencies

Reciprocity estimation and enforcement

- Example of S-parameters of reciprocal 4-port interconnect (symmetric matrix):



$$S = \begin{bmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{1,2} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{1,3} & S_{2,3} & S_{3,3} & S_{3,4} \\ S_{1,4} & S_{2,4} & S_{3,4} & S_{4,4} \end{bmatrix}$$

Reciprocity measure can be computed as mean difference between elements that have to be equal (at each frequency point):

$$RM = \frac{1}{N_s} \sum_{i,j} \left| S_{i,j} - S_{j,i} \right|$$

or max singular value of $S - S^t$ can be used

RM is compared with a *threshold*: if *RM* > *threshold*, the multiport is reported as not reciprocal

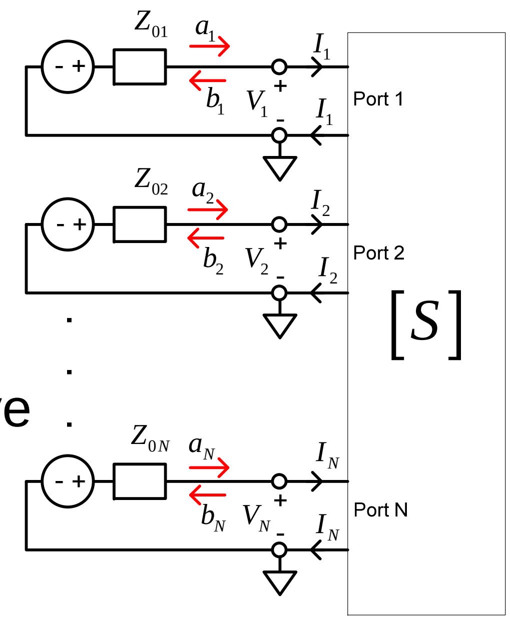
Averaging can be used to "enforce" the reciprocity (works only with noisy data):

$$S_{j,i} = S_{i,j} = 0.5\left(S_{i,j} + S_{j,i}\right)$$

Passivity

- Power transmitted to multiport is a difference of power transmitted by incident and reflected waves: $P_{in} = \sum_{n=1}^{N} |a_n|^2 - |b_n|^2 = \left[\overline{a}^* \cdot \overline{a} - \overline{b}^* \cdot \overline{b}\right]$

  or $P_{in} = \overline{a}^* \cdot \overline{a} - \overline{a}^* \cdot S^* S \cdot \overline{a} = \overline{a}^* \cdot \left[U - S^* S\right] \cdot \overline{a}$



- Transmitted power is defined by Hermitian quadratic form and must be not negative for passive multiport for any combination of incident waves

- Quadratic form is non-negative if eigenvalues of the matrix are non-negative (Golub & Van Loan):

$$eigenvals\left[U - S^* \cdot S\right] \geq 0 \Rightarrow eigenvals\left[S^* \cdot S\right] \leq 1$$

(*U* is unit matrix)

Sufficient condition only if verified from DC to infinity

More on passivity

- Maximal singular value of S can be used for passivity estimation, because of non-zero singular values of S are square roots of eigenvalues of *S*S* (Golub & Van Loan)

$$\delta_i = \sqrt{\lambda_i}, \quad \lambda_i = eigenvals\left(S^* \cdot S\right) \; \lambda_i \in R, \lambda_i \geq 0$$

- Passivity of symmetric S can be estimated with eigenvalues as $\left|eigenvals\left(S\right)\right| \leq 1$
  - It is possible due to the fact that singular values of symmetric matrices are equal to the magnitudes of the eigenvalues

- Common mistake is to estimate passivity as:

$$\sum_{k=1}^{N} \left|S_{i,k}\right|^2 \leq 1 \qquad \text{or} \qquad \left|S_{i,k}\right| \leq 1$$

This is necessary but not sufficient condition!

Passivity estimation and enforcement

- Passivity conditions for S-parameters (energy dissipation condition):

$$eigenvals(U - S^* \cdot S) \geq 0 \quad \Rightarrow \quad eigenvals(S^* \cdot S) \leq 1$$

Passivity measure is computed at each frequency point as:

$$PM = \sqrt{\max\left[eigenvals(S^* \cdot S)\right]}$$

is equal to max singular value of S

PM is compared with a *threshold*: if *PM* > *threshold*, the multiport is reported as not passive

Normalization at each frequency point can be used to "enforce" the passivity (works only with minor violations):

$$if \;\; PM > 1.0 \Rightarrow S_p = \frac{S}{PM}$$
$$else \; S_p = S$$

Alternatively a rational filter can be used

Causality in frequency-domain

- Condition $H(t) = 0 \; at \; t < 0$ for the unit pulse response matrix and

$$H(i\omega) = \int_{-\infty}^{\infty} H(t) \cdot e^{-i\omega t} \cdot dt, \quad H(i\omega) \in C^{N \times N}$$

  leads to Kramers-Kronig relations for the frequency-domain parameters (Hilbert transform)

$$H(i\omega) = \frac{1}{i\pi} PV \int_{-\infty}^{\infty} \frac{H(i\omega')}{\omega - \omega'} \cdot d\omega', \quad PV = \lim_{\varepsilon \to 0} \left(\int_{-\infty}^{\omega - \varepsilon} + \int_{\omega + \varepsilon}^{+\infty} \right)$$

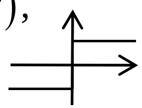
$$H_r(\omega) = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{H_i(\omega')}{\omega - \omega'} \cdot d\omega', \quad H_i(\omega) = \frac{-1}{\pi} PV \int_{-\infty}^{\infty} \frac{H_r(\omega')}{\omega - \omega'} \cdot d\omega'$$

Kramers, H.A., Nature, v 117, 1926 p. 775..
Kronig, R. de L., J. Opt. Soc. Am. N12, 1926, p 547.

derivation

$$H(t) = sign(t) \cdot H(t),$$

$$sign(t) = \begin{cases} -1, & t < 0 \\ 1, & t > 0 \end{cases}$$



$$H(i\omega) = F\{H(t)\} =$$

$$= \frac{1}{2\pi} F\{sign(t)\} * F\{H(t)\}$$

$$F\{sign(t)\} = \frac{2}{i\omega}$$

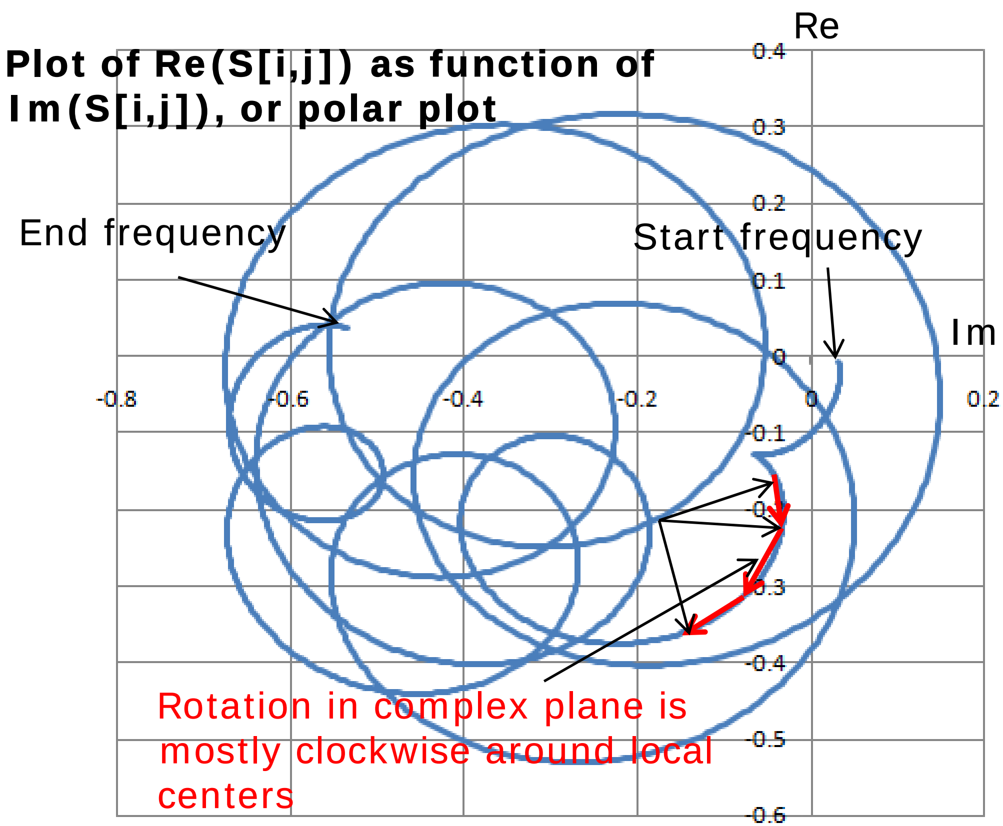
- Imaginary part can be derived from real (or vice versa), but the other part must be known from DC to infinity

Causality estimation - difficult way

- Kramers-Kronig relations cannot be directly used for the frequency-domain response known over the limited bandwidth
- Causality boundaries can be introduced to estimate causality of the tabulated and band-limited data sets
  - Milton, G.W., Eyre, D.J. and Mantese, J.V, *Finite Frequency Range Kramers Kronig Relations: Bounds on the Dispersion,* Phys. Rev. Lett. 79, 1997, p. 3062-3064
  - Triverio, P. Grivet-Talocia S., *Robust Causality Characterization via Generalized Dispersion Relations*, IEEE Trans. on Adv. Packaging, N 3, 2008, p. 579-593.

Causality estimation - easy way

- “Heuristic” causality measure based on the observation that polar plot of a causal system rotates mostly clockwise (suggested by V. Dmitriev-Zdorov)



Causality measure (CM) can be computed as the ratio of clockwise rotation measure to total rotation measure in %.

If this value is below 80%, the parameters are reported as suspect for possible violation of causality.

Causality improvement

- Filtration or decimating – the simplest technique, but may further degrade the response quality
- Artificially extend real or imaginary part, or magnitude of the frequency response to DC and to the infinity and restore the other part with the Kramers-Kronig equations
  - The restored part will strongly depend on the artificial extension
  - Iterative extension adjustment is possible to improve accuracy over the sampled frequency band - difficult to implement
- Fit the response with causal rational basis functions (use rational compact model)
  - Provides controlled accuracy over the sampled frequency band
  - Consistent results in both frequency and time domains
  - Can be extended to DC and to infinity

Use of Rational Compact Model (RCM) for S-parameters causality "improvement"

$$\bar{b} = S \cdot \bar{a}, \quad S_{i,j} = \left.\frac{b_i}{a_j}\right|_{a_k=0\; k\neq j} \Rightarrow S_{i,j}(i\omega) = \left[d_{ij} + \sum_{n=1}^{N_{ij}} \left(\frac{r_{ij,n}}{i\omega - p_{ij,n}} + \frac{r^*_{ij,n}}{i\omega - p^*_{ij,n}} \right) \right] \cdot e^{-s\cdot T_{ij}}$$

Continuous functions of frequency

$s = i\omega$, d_{ij} – values at ∞, N_{ij} – number of poles,

$r_{ij,n}$ – residues, $p_{ij,n}$ – poles (real or complex), T_{ij} – optional delay

- Pulse response is real and delay-causal

$$S_{i,j}(t) = 0, \quad t < T_{ij}$$

$$S_{i,j}(t) = d_{ij}\delta(t - T_{ij}) + \sum_{n=1}^{N_{ij}} \left[r_{ij,n} \cdot \exp\left(p_{ij,n} \cdot (t - T_{ij}) \right) + r^*_{ij,n} \cdot \exp\left(p^*_{ij,n} \cdot (t - T_{ij}) \right) \right], \quad t \geq T_{ij}$$

- Stable $\mathrm{Re}(p_{ij,n}) < 0$
- Passive if $eigenvals\left[S(\omega) \cdot S^*(\omega) \right] \leq 1 \ \forall \omega, \ from\ 0\ to\ \infty$
- Reciprocal if $S_{i,j}(\omega) = S_{j,i}(\omega)$

May require enforcement (passivity and reciprocity)

What are RCMs for?

- Improve quality of tabulated Touchstone models
  - Fix minor passivity and causality violations
  - Interpolate and extrapolate with guarantied passivity
- Produce broad-band SPICE models
  - Much smaller model size
  - No artifacts and guarantied stability of SPICE simulation
  - Consistent frequency and time domain analyses
- Compute time-domain response of a channel with a fast recursive convolution algorithm (exact solution for PWL signals)

Global quality metrics (0-100%)

- Passivity Quality Measure: PQM or zero if PQM<0

$$PQM = \frac{100}{N_{total}}\left[N_{total} - \sum_{n=1}^{N_{total}} PW_n\right]\% \qquad PW_n = 0 \; if \;\; PM_n < 1.00001; \; otherwise \; PW_n = \frac{PM_n - 1.00001}{0.1}$$

$$PM_n = \sqrt{\max\left[eigenvals\left(S^*(f_n)\cdot S(f_n)\right)\right]}$$

should be >98%

- Reciprocity Quality Measure: RQM or zero if RQM<0

$$RQM = \frac{100}{N_{total}}\left[N_{total} - \sum_{n=1}^{N_{total}} RW_n\right]\% \qquad RW_n = 0 \; if \;\; RM_n < 10^{-6}; \; otherwise \; RW_n = \frac{RM_n - 10^{-6}}{0.1}$$

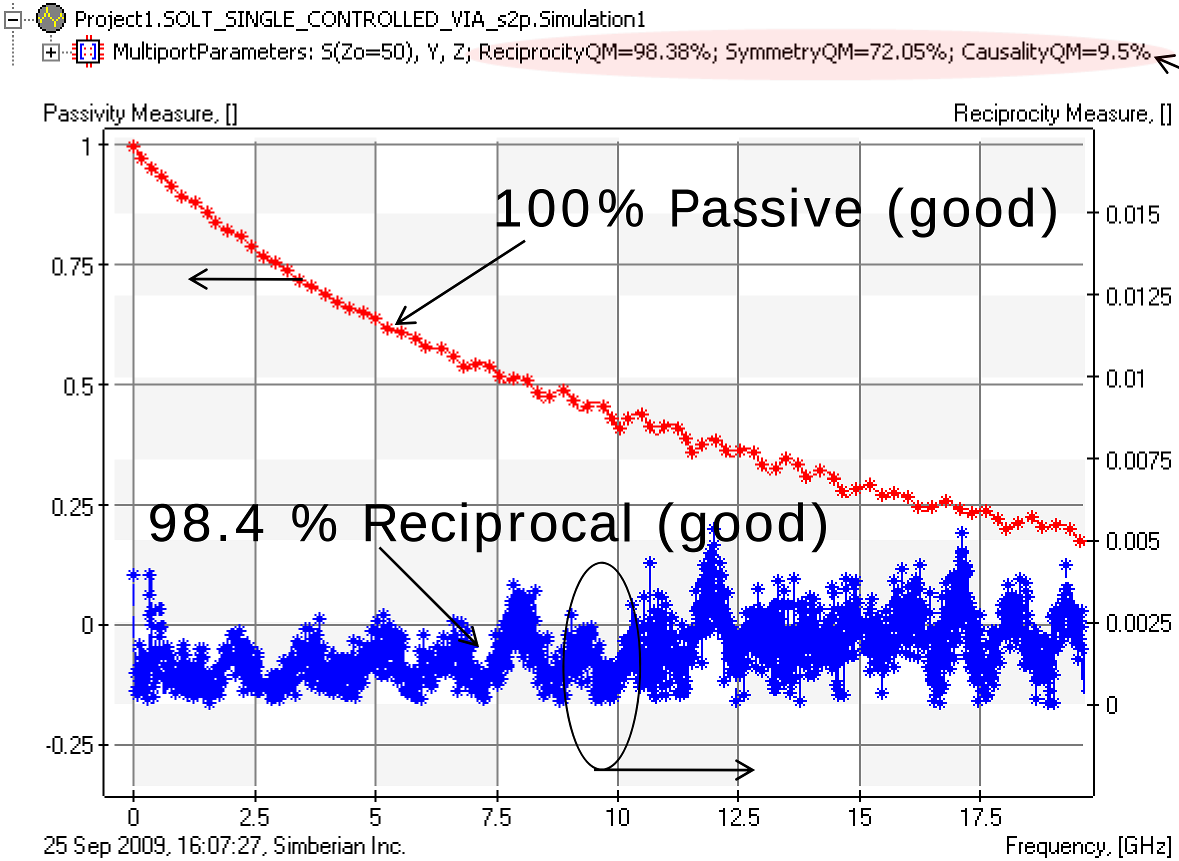
$$RM_n = \frac{1}{N_s}\sum_{i,j}\left|S_{i,j}(f_n) - S_{j,i}(f_n)\right|$$

should be >98%

- Causality Quality Measure: Minimal ratio of clockwise rotation measure to total rotation measure in % (should be >80%)
- RMS error of the rational compact model can be also used to characterize the causality of the original data set

Example 1: High-quality model

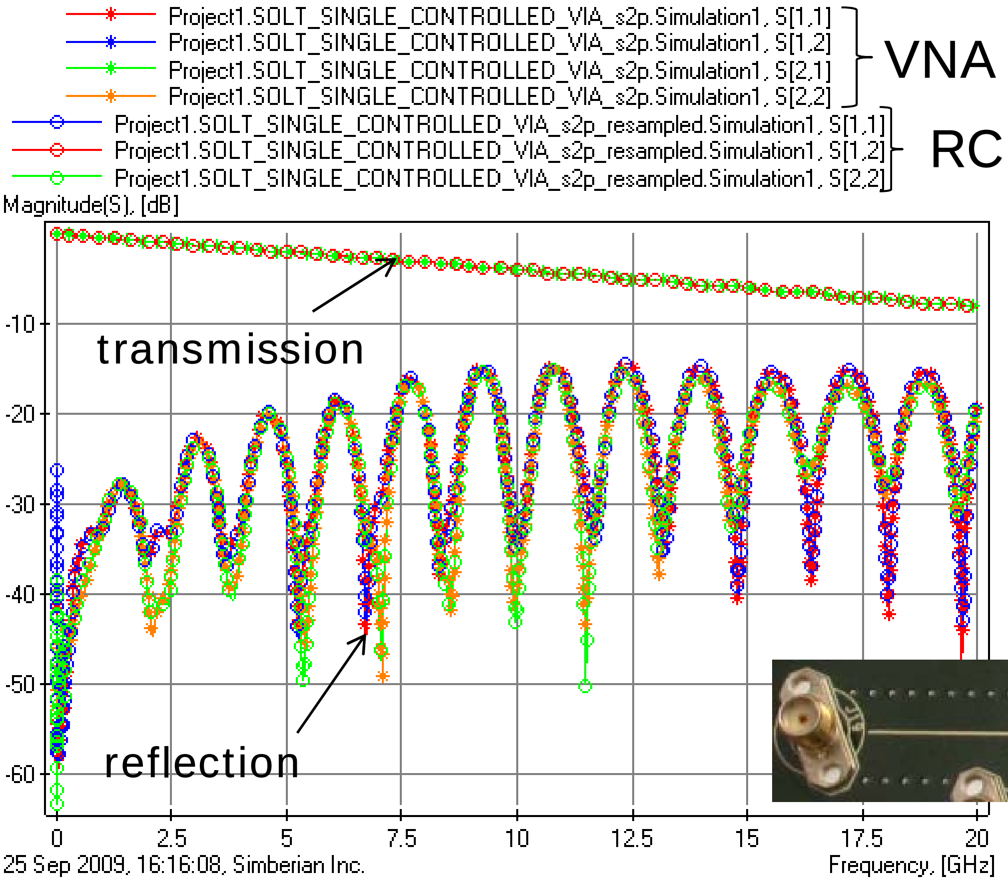
Single controlled via from PLRD-1 benchmark board – SOLT calibration



Causality problem, but it can be restored with RCM

Data provided by Teraspeed Consulting Group

Single controlled via (SOLT): Improving S-parameters with RCM



RCM model has RMS Error is 0.0034 (very good), is passive from DC to infinity and 100% causal and reciprocal

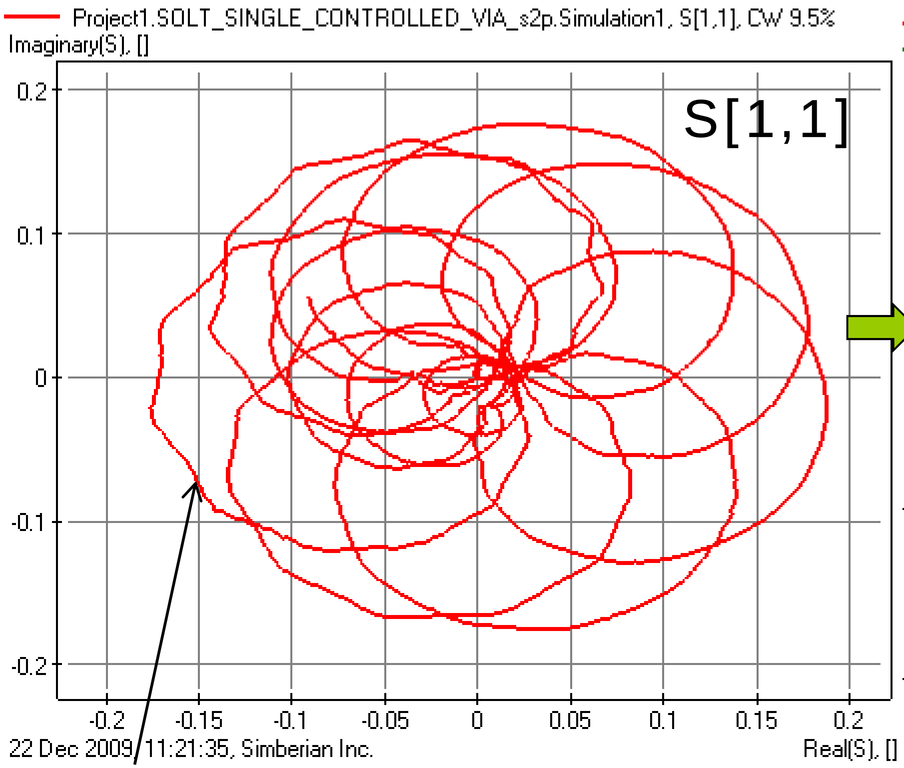
Touchstone model with DC and reduced number of frequency points or BB SPICE model can be produced from RCM

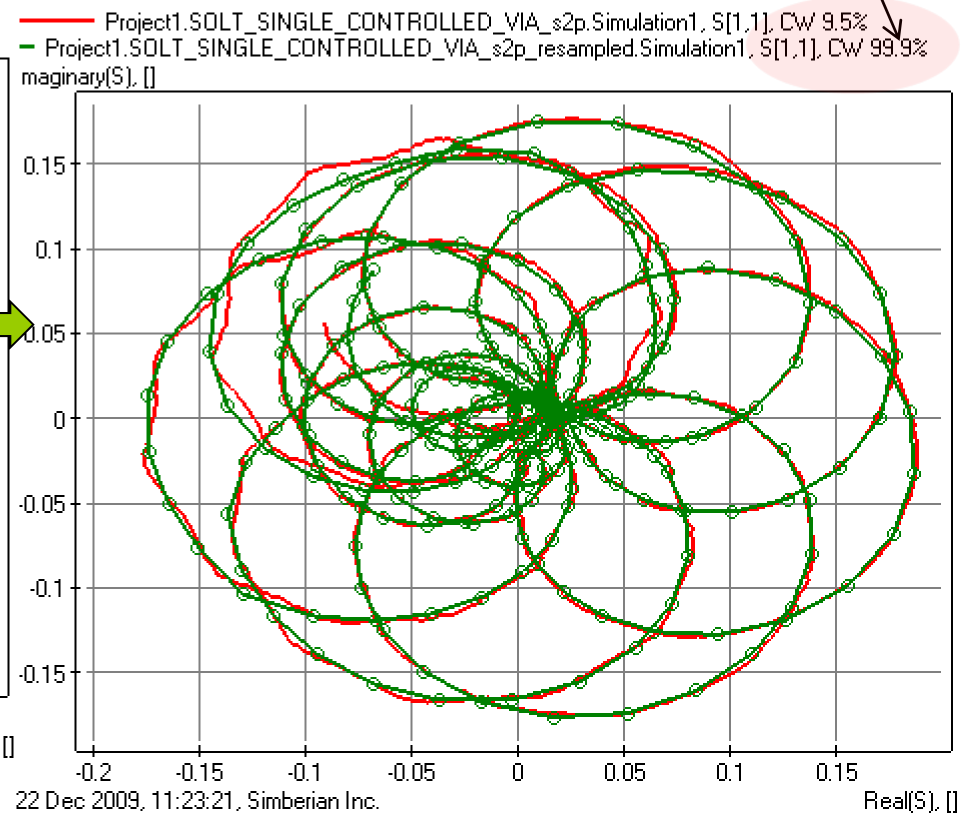
Single controlled via (SOLT): Original S[1,1] and RCM

VNA Measurement: 3201 points starting from 300 KHz

Re-sampled RCM: 769 points distributed adaptively starting from 0 Hz

CAUSAL!

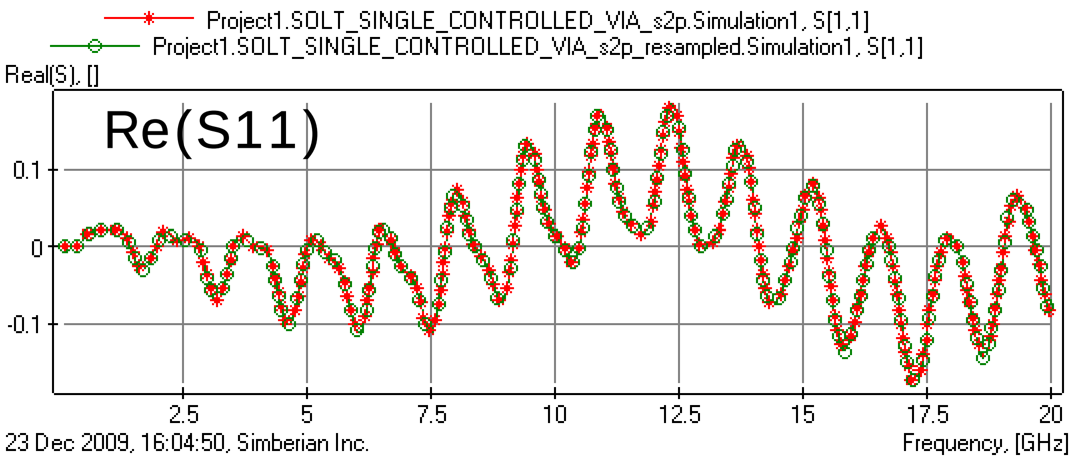


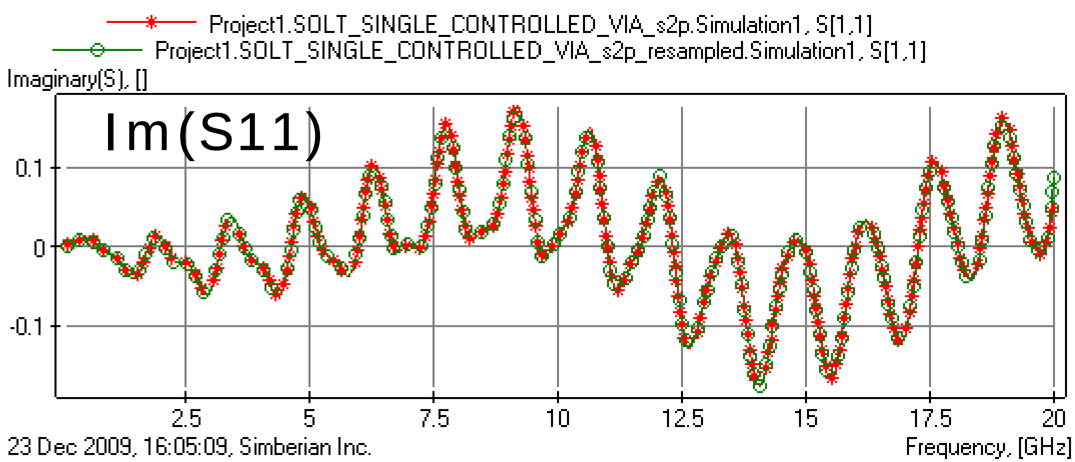


Visible noise and large segments with counter-clockwise rotation

Red line – original VNA
Green line with circles - RCM

Single controlled via (SOLT): Original S[1,1] and RCM





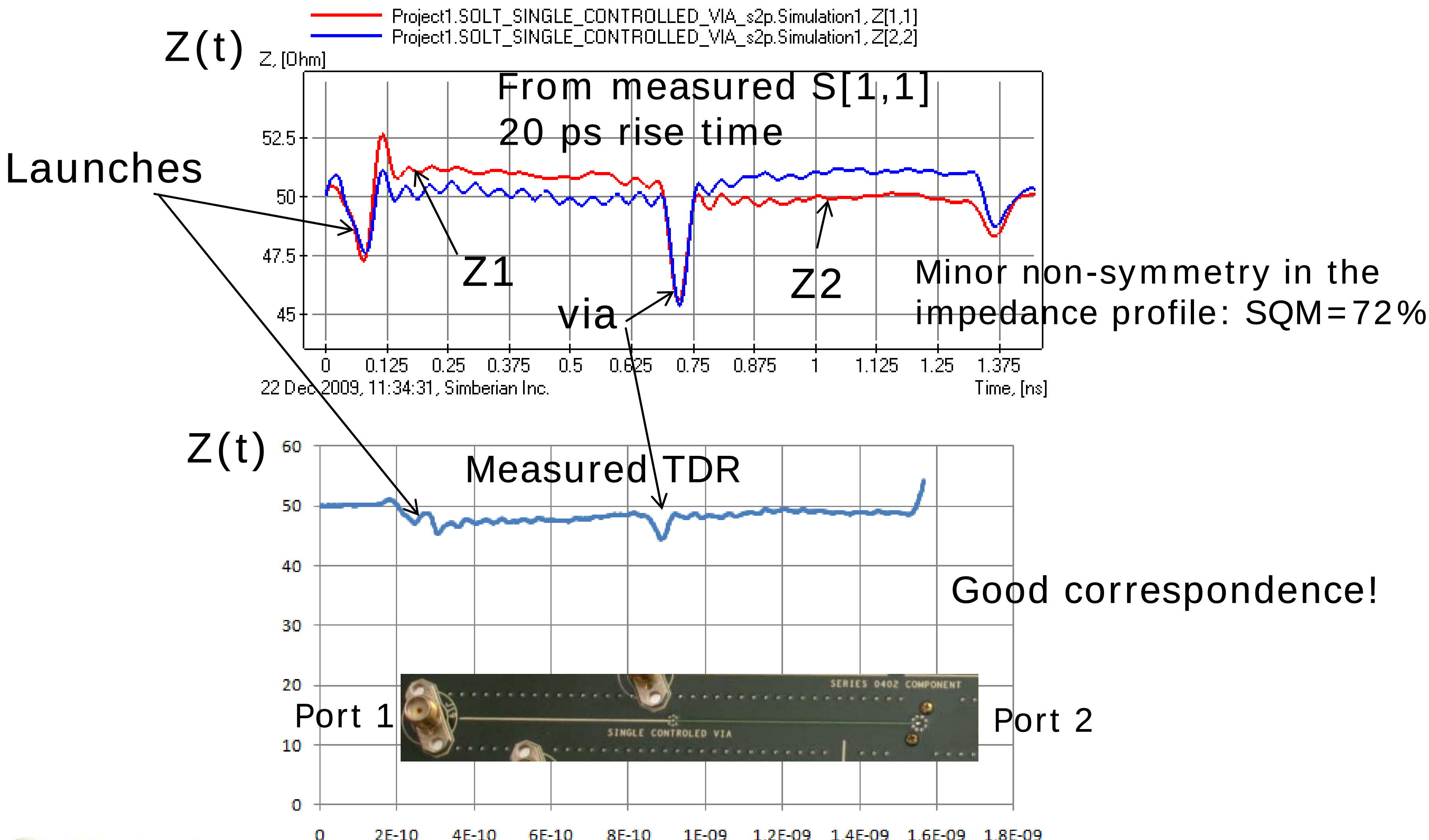
Stars – VNA data

Circles – RCM

RCM: 46 poles,
RMS Error 0.0034

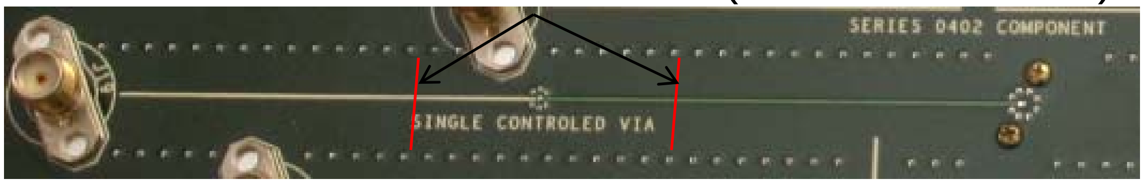
Practically indistinguishable!

Single controlled via TDR from RCM (SOLT)

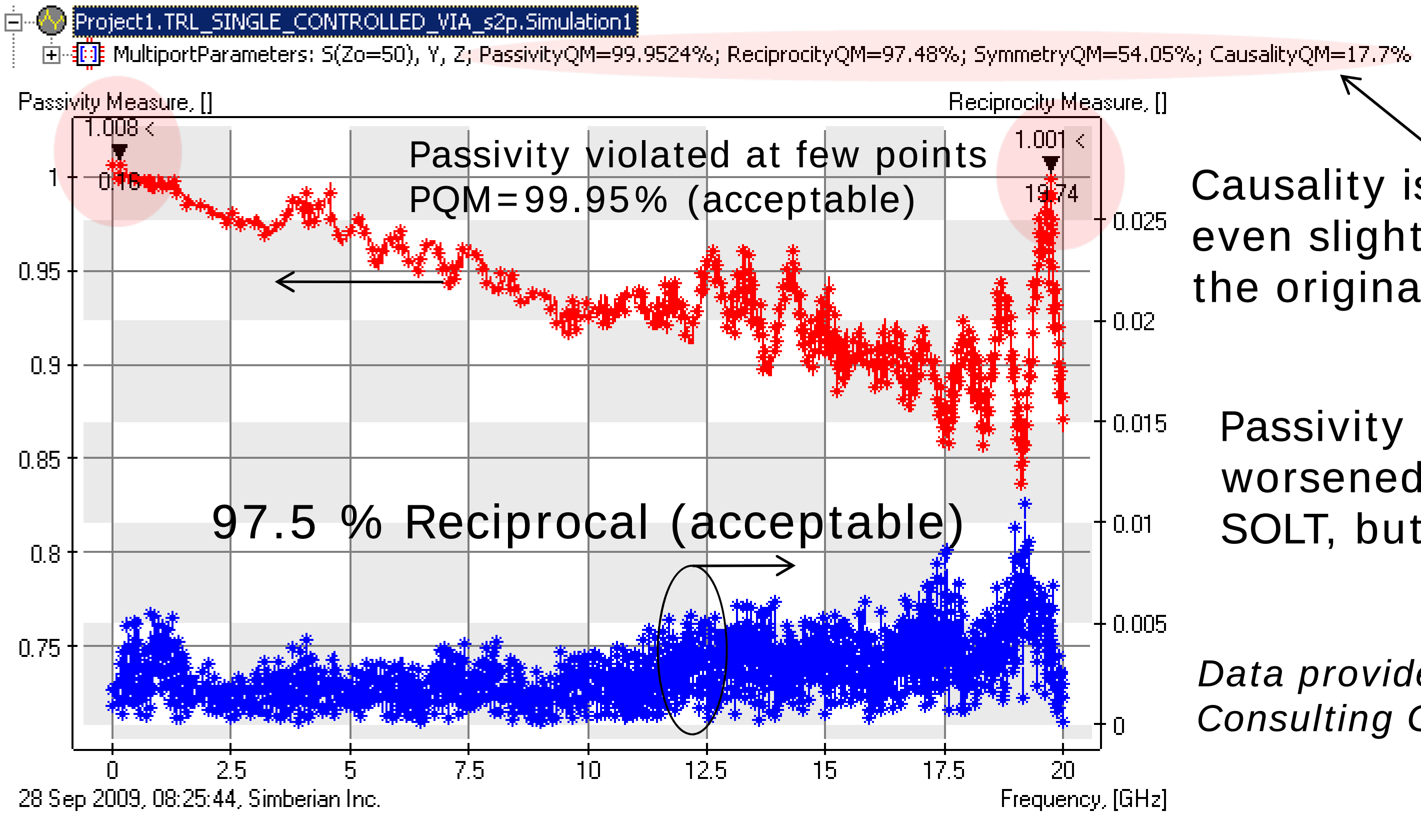


Example 2: Model that needs improvement

TRL Reference Planes (250 mil from via)



Single controlled via from PLRD-1 benchmark board – TRL calibration



Causality is 17.7%, that is even slightly better than the original SOLT 9.5%

Passivity and reciprocity worsened comparing to SOLT, but still OK

Data provided by Teraspeed Consulting Group

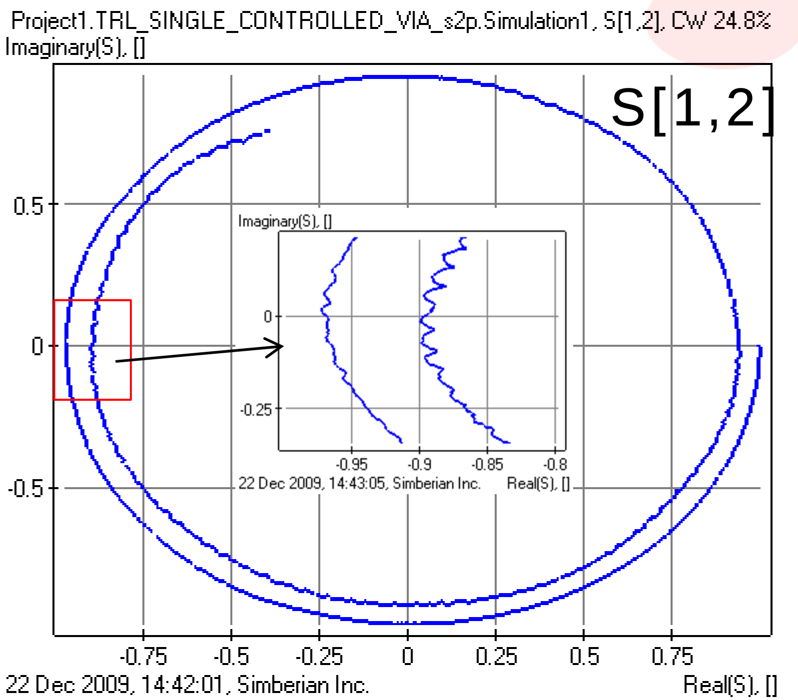
Single controlled via (TRL): Causality problems both in transmission and reflection

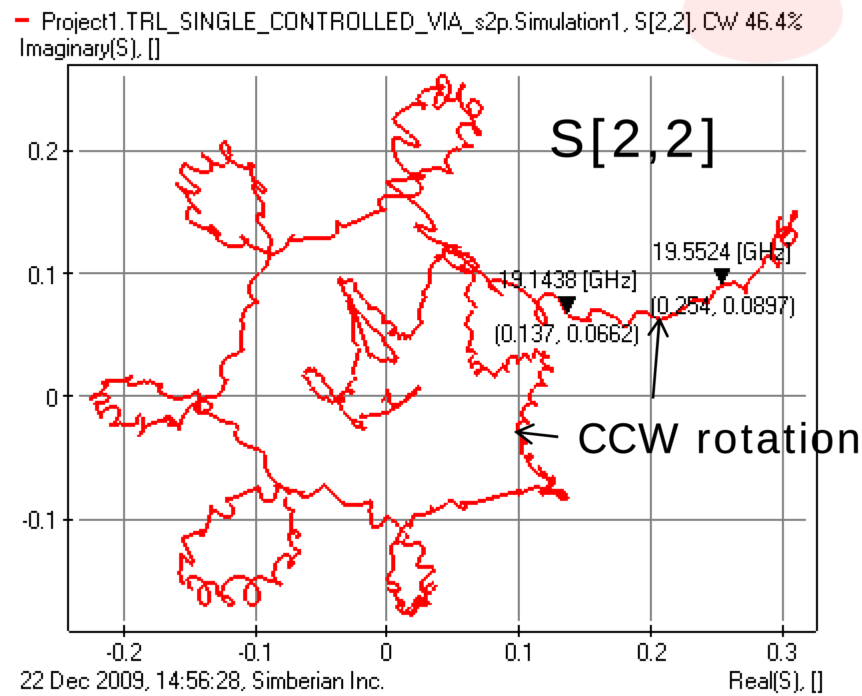
TRL Reference Planes (250 mil from stubs)

Port 1

Port 2

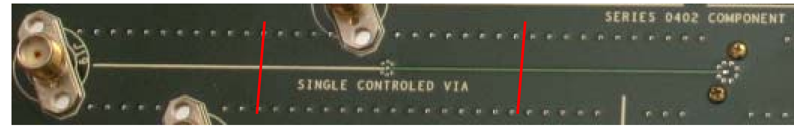
Some problems both in the transmission and reflection parameters (can be fixed):



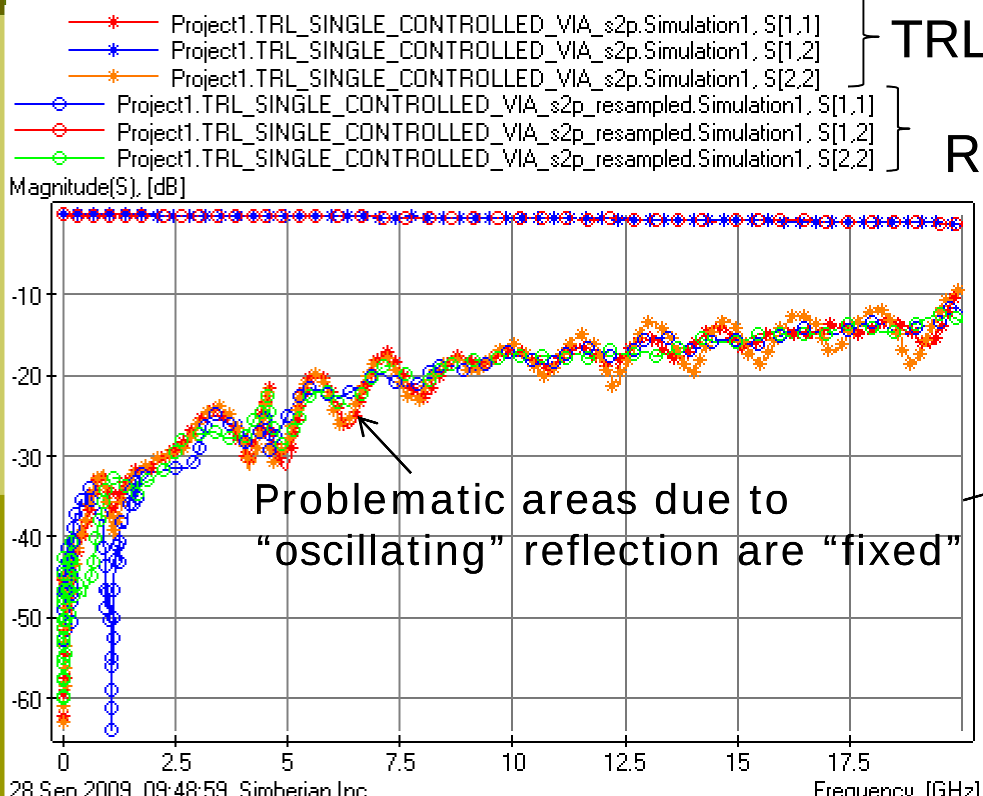


Single controlled via (TRL): Improving S-parameters with RCM

RCM RMS Error is 0.045 (still OK)
Passive from DC to infinity, causal and reciprocal



Transmission and group delay is OK

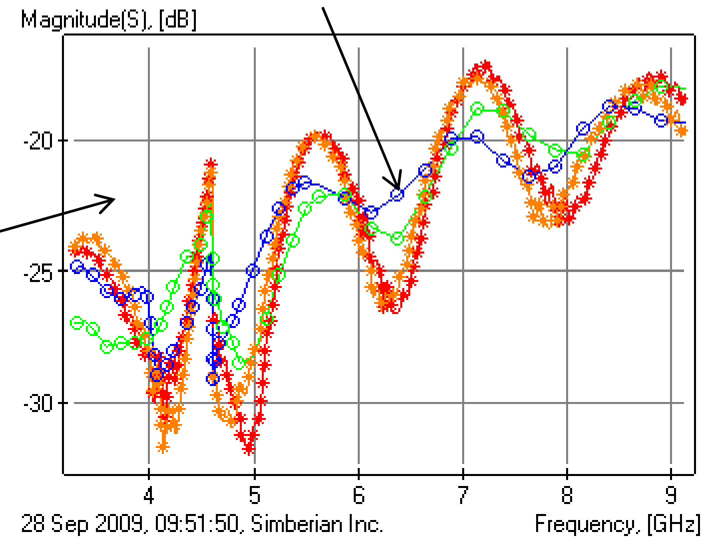


TRL

RCM

Problematic areas due to "oscillating" reflection are "fixed"

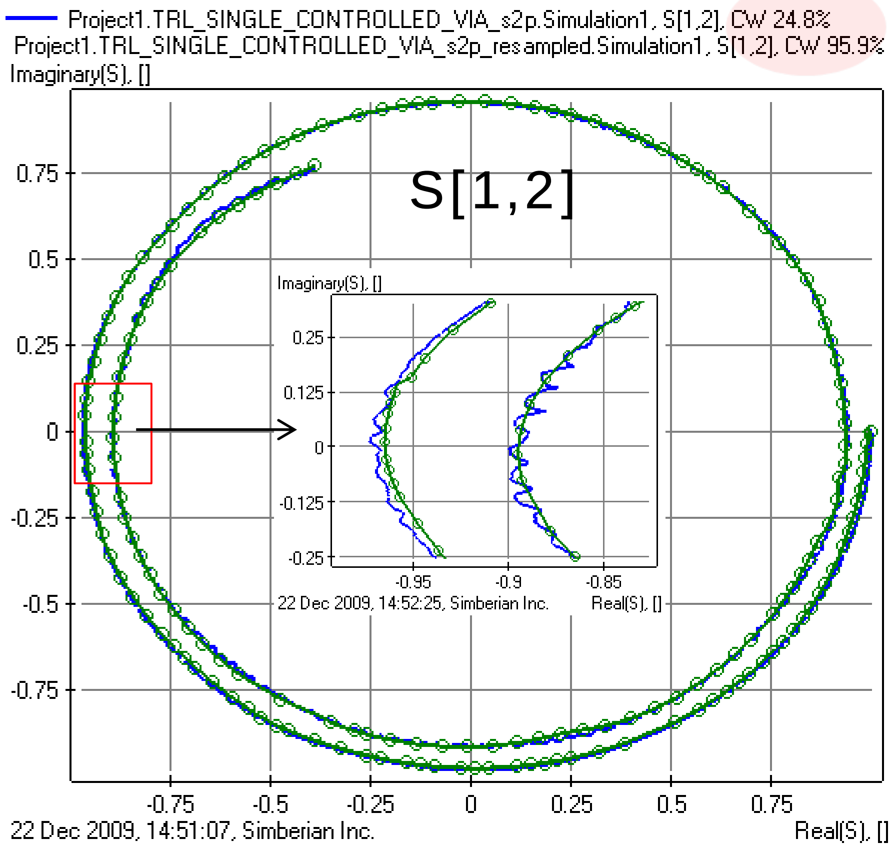
Problem is in the reflection parameters and RCM "fixes" it with the best possible fit



Single controlled via (TRL): Original S[1,2] and RCM

VNA Measurement: 3201 points starting from 300 KHz

Re-sampled RCM: 633 points distributed adaptively starting from 0 Hz

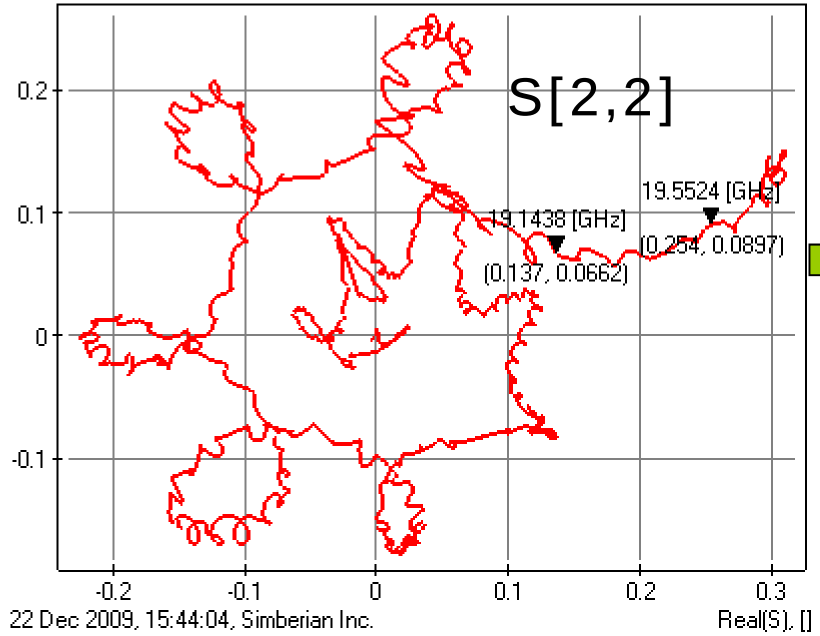


CAUSAL!

Very noisy data is corrected with RCM!

Single controlled via (TRL): Original S[2,2] and RCM

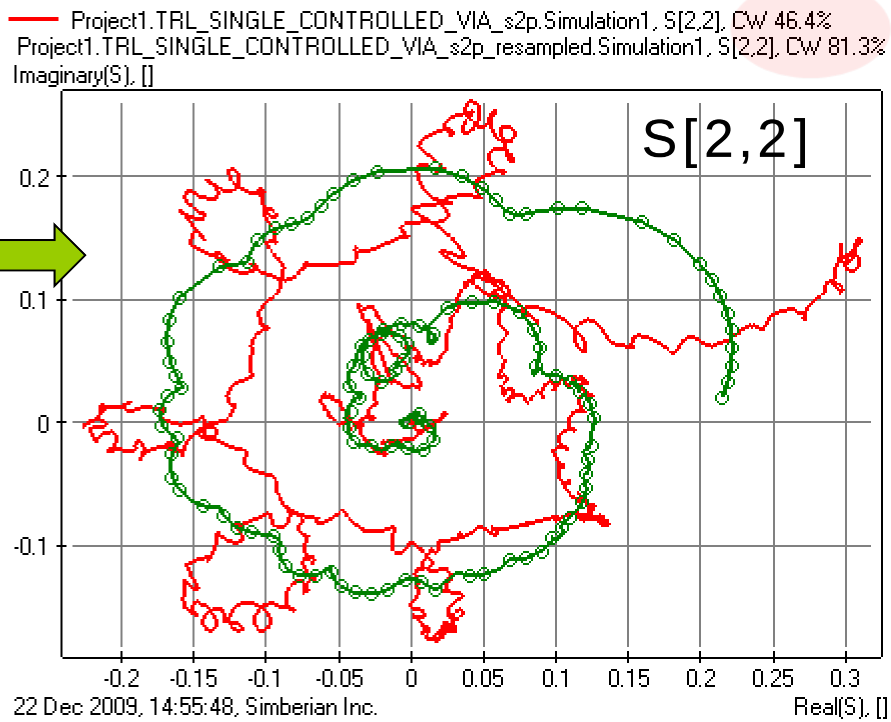
VNA Measurement: 3201 points starting from 300 KHz



Very noisy non-causal data with wrong rotation!

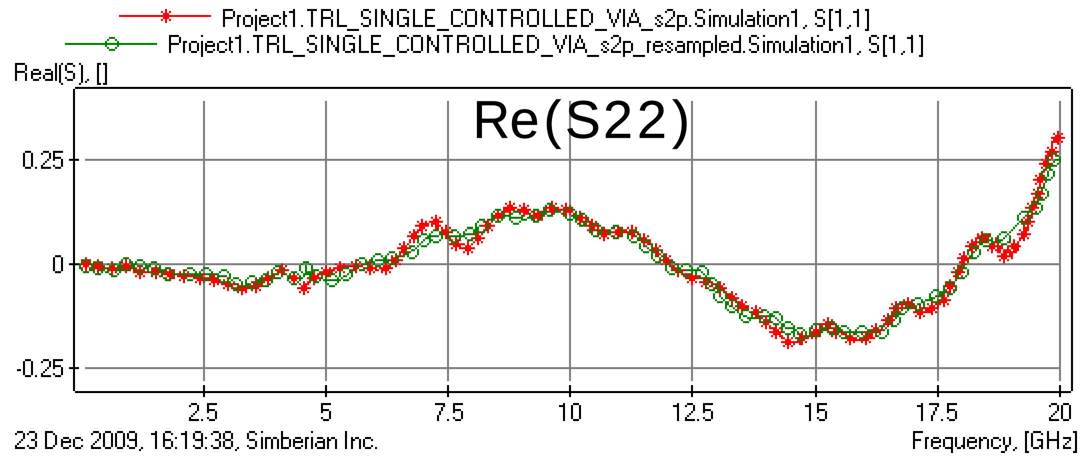
Re-sampled RCM: 633 points distributed adaptively starting from 0 Hz

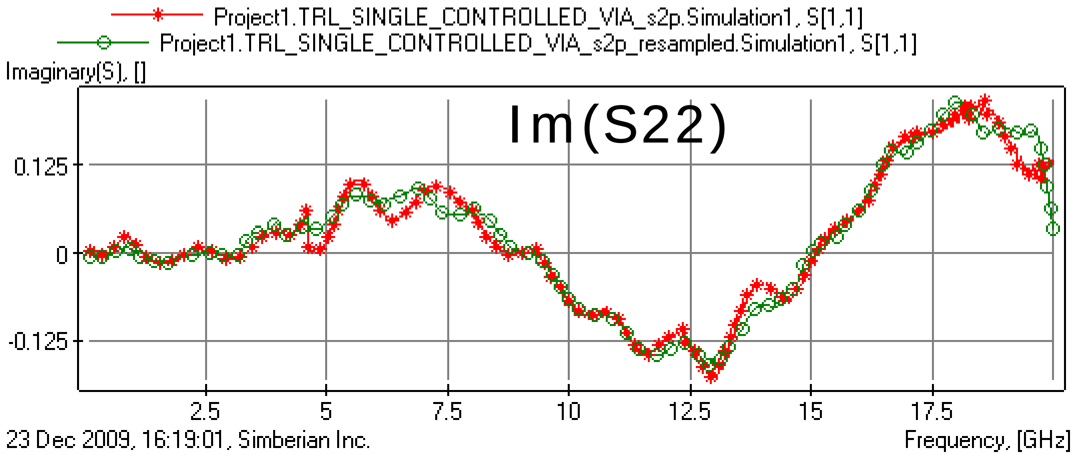
Does not match well but CAUSAL ☺



Red line – original TRL data
Green line with circles - RCM

Single controlled via (TRL): Original S[2,2] and RCM





Stars – original TRL data

Circles – RCM model

RMS Error 0.045,
44 poles

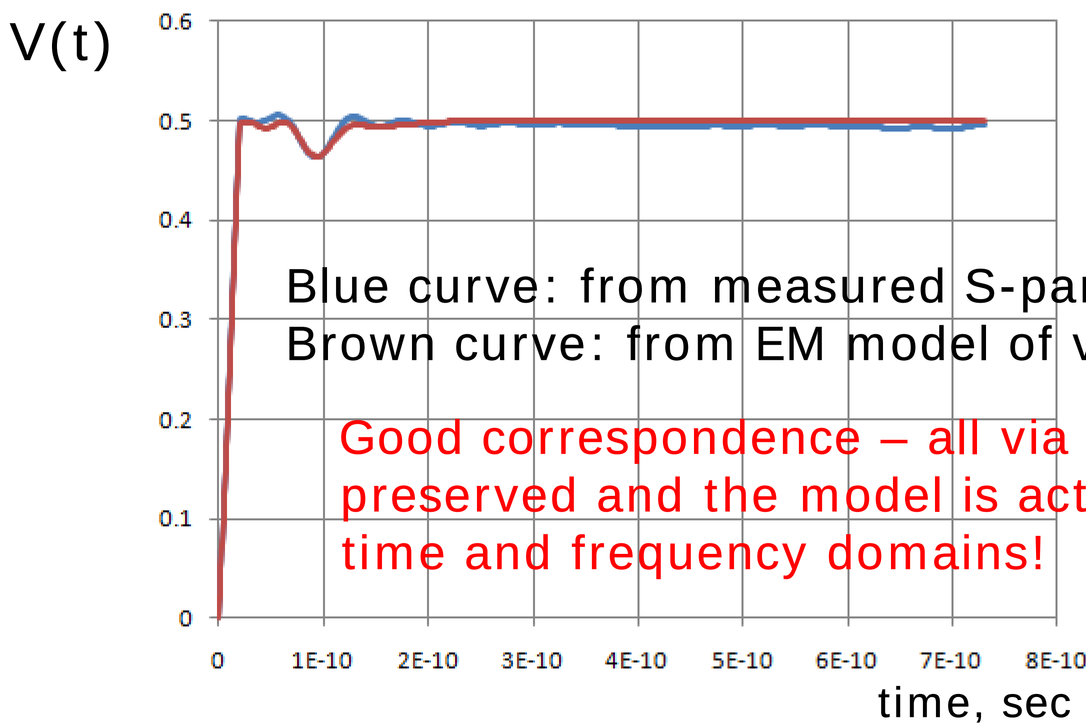
Problematic non-causal areas are fitted as close as possible

Does the corrected data contain information about the via?

Single controlled via TDR from RCM (TRL)

Pure via in a t-line:
no connector and launch
discontinuities

TRL Reference Planes (250 mil from stubs)



Blue curve: from measured S-parameters
Brown curve: from EM model of via

Good correspondence – all via properties are preserved and the model is actually usable in the time and frequency domains!

Conclusion

- Reciprocity, passivity and causality of interconnect component models must be verified before use
  - Measured models may be not acceptable for the analysis
  - Electromagnetic models may have severe problems too
- Quality metrics allow distinguishing minor “fixable” violations with acceptable accuracy degradation from severe violations
- Rational macro-models with controllable accuracy can be used to “improve” tabulated models and to correct minor violations of passivity and causality
- Standardization of the quality metrics and exchange formats for rational compact models are needed

Contact and resources

- Yuriy Shlepnev, Simberian Inc.

  shlepnev@simberian.com

  Cell: 206-409-2368
- Free version of Simbeor 2008.L0 used to plot and estimate quality of S-parameters is available at www.simberian.com
- To learn on quality metrics further see slides from DesignCon2010 tutorial (also available on request)
  - TF-MP12 H. Barnes, Y. Shlepnev, J. Nadolny, T. Dagostino, S. McMorrow, Quality of High Frequency Measurements: Practical Examples, Theoretical Foundations, and Successful Techniques that Work Past the 40GHz Realm