How Interconnects Work: Characteristic Impedance and Reflections

Yuriy Shlepnev, Simberian Inc.

Analysis of "digital interconnects" is the analog problem in frequency domain where interconnects are simulated as transmission lines defined by characteristic impedance and propagation constant. Digital signals in interconnects are sequences of amplitude-modulated pulses that transmit bits between components. The "digital interconnect" analysis problem is technically an analog problem of pulse propagation modeling in time-domain. Sequence of the transmitted bits (1s and 0s) is the only boundary between the digital and the analog interconnect analysis domains. **Though, that time-domain analysis problem is practically always solved in the frequency domain.** Pulse or sequence of pulses are transformed into a superposition of harmonics or sinusoidal signals in time domain (more on that in [1]) because of **it is mathematically easier and more convenient to model all types of signal degradation for the harmonic signals using phasors and complex analysis.** Components on PCBs in the digital domain are just connected – 1s and 0s are supposed to flow seamlessly between the components. In the analog or RF/microwave domain, **components on PCBs or in a package are connected with the distributed open waveguiding structures** composed of traces and reference conductors and simulated mostly as transmission lines. To ensure that the digital signal is actually getting through, we have to build interconnect models that include all signal degradation factors important for a specific data rate.

In general all signal degradation factors can be separated into 3 categories:

- **Absorption losses in dielectrics and conductors;**
- **Reflection losses due to impedance mismatch and discontinuities;**
- **Coupling losses and distortion (includes crosstalk);**

The absorption or dissipation losses in dielectrics and conductors were recently discussed in [2]. Such losses are inevitable, but can be effectively mitigated at the stackup planning stage – selection of dielectric and conductor materials and stackup geometry defines the maximal possible communication distance for a particular data rate.

Considering the reflections, they can be further separated into the following 3 categories:

- **Reflections from transmission lines and termination impedance mismatch;**
- **Reflections from single discontinuities – vias, transitions, AC caps, gaps in reference plane,…;**
- **Reflections from periodic discontinuities – cut outs, fiber-weave effect,…;**

Why do we care about the reflections? – Because of reflections degrade the transmitted signal and such degradation may cause link failure. Thus, understanding and evaluation of reflections is useful for channel quality control and there are corresponding compliance metrics in frequency domain (bounds on reflection loss) as well as in time domain (Effective Return Loss or ERL). **Here we will take a closer look at the reflections caused by the transmission line characteristic impedance and termination impedance mismatch.** We have discussed it in our "Design Insights…" tutorial at the last "normal" DesignCon in 2020 [3] and this paper is loosely based on that.

Impedance and admittance as well as impedivity, admittivity, conductance (conductivity?), succeptance, leakance, voltivity, gaussivity are the terms introduced by Oliver Heaviside at the end of 19th century during the golden era of electromagnetic discoveries started by James Clerk Maxwell. Heaviside derived the Telegrapher's equations describing transmission lines or, as we know now, any waveguiding system in general. The equations describe 1-dimensional distributed problem that for a 2-conductor or 1-mode (one signal and one reference conductor) transmission line looks as follows:

$$\frac{\partial V(x)}{\partial x} = -Z(f)\cdot I(x) \qquad Z(f) = R(f) + i2\pi f \cdot L(f)$$

$$\frac{\partial I(x)}{\partial x} = -Y(f)\cdot V(x) \qquad Y(f) = G(f) + i2\pi f \cdot C(f)$$

Where *I* is the current, *V* is voltage changing along the x-axis, *f* is frequency [Hz]. *Z* [Ohm/m] is complex impedance per unit length and *Y* [S/m] is complex admittance per unit length, R [Ohm/m] and L [Hn/m] – are real frequency-dependent resistance and inductance per unit length, G [S/m] and C [F/m] – are real frequency-dependent conductance and capacitance per unit length. *Z, Y, R, L, G, C* for N+1 conductor problem or N-mode transmission line are NxN matrices in general. They are 2x2 matrices for a 3-conductor differential line for instance. The impedance and admittance per unit length are frequency-dependent in general and are completely defined by transmission line type and cross-section and usually computed either with a static or quasi-static 2-D field solver or sometime with 3D EM solvers. Note that the use of 3-D solvers does not automatically guarantee higher accuracy.

A solution of the Telegrapher's equation can be written as a superposition of two waves propagating in opposite directions with as follows (can be easily verified by inspection):

$$V(x) = v^{+}\cdot\exp(-\Gamma\cdot x) + v^{-}\cdot\exp(\Gamma\cdot x)$$

$$I(x) = \frac{1}{Z_c}\left[v^{+}\cdot\exp(-\Gamma\cdot x) - v^{-}\cdot\exp(\Gamma\cdot x)\right]$$

$$\Gamma(f) = \sqrt{Z(f)\cdot Y(f)} = \alpha(f) + i\beta(f)$$

$$Z_c(f) = \sqrt{Z(f)/Y(f)}$$

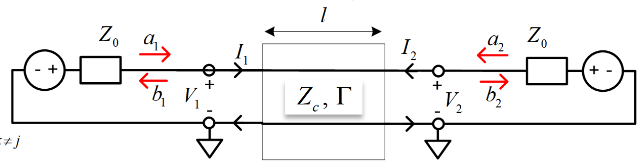
Where *Zc* is complex frequency-dependent **characteristic impedance** and Gamma is complex **propagation constant** (alfa is the attenuation constant [Np/m] and beta is the phase constant [rad/m] defined as 2*PI/Lambda, Lambda is the wavelength in the transmission line – phase changes by 2*PI over that length, see more in the Appendix). Those are the modal parameters in general – the equations above are for a 2-conductor line with 1 mode only. If we write the solution for the wave propagating only in one direction along the x-axis for instance (would be ideal for signal transmission):

$$v(x) = v^{+}\cdot\exp(-\Gamma\cdot x), \quad i(x) = \frac{v^{+}}{Z_c}\exp(-\Gamma\cdot x)$$

We can see that the **characteristic impedance** is just a ratio of the voltage and current of the wave propagating in one direction of transmission line *v(x)/i(x)=Zc*. It is impedance by dimension [Ohm]. It is pure resistance if line is lossless. Word "characteristic" is used because of it does not depend on the position or length of transmission line segment (independent of x) – it "characterizes" it. It depends only on the type of transmission line and geometry of the cross-section. Note that for planar transmission lines, used for PCB and packaging interconnects, the definition of impedance is not unique and can be done in 3 ways – through voltage and current, current and power and voltage and power, but they all close to the conventional "static" voltage-current definition if cross-section remains much smaller than the wavelength, which is usually good assumption for PCB and packaging interconnects.

To investigate the reflections, the next step is to define properties of a transmission line segment. The Telegrapher's equations introduced in the previous section are incomplete without the "boundary conditions" or terminations. The most effective way to describe a segment is to use waves and scattering parameters or S-parameters. Here is a transmission line segment with length *l* connected to voltage sources with all variables, to define S-parameters:

$$S_{i,j} = \left.\frac{b_i}{a_j}\right|_{a_k=0\ k\neq j}$$



Incident Waves: $a_1 = \frac{1}{2\sqrt{Z_0}}\left(V_1 + Z_0 \cdot I_1\right)$ $\quad a_2 = \frac{1}{2\sqrt{Z_0}}\left(V_2 + Z_0 \cdot I_2\right)$

Reflected Waves: $b_1 = \frac{1}{2\sqrt{Z_0}}\left(V_1 - Z_0 \cdot I_1\right)$ $\quad b_2 = \frac{1}{2\sqrt{Z_0}}\left(V_2 - Z_0 \cdot I_2\right)$

Where *a1, a2* are the "incident waves", and *b1, b2* are the "reflected waves" with dimension sqrt(Wt). *V1,V2* and *I1, I2* are voltages and currents at the segment ports (pairs of terminals). Zo is the termination or normalization impedance (same thing in this context). Waves in this definition are not actual waves in the transmission line, but rather variables formally defined through voltage and current. Using equations for voltage and current in transmission line segment (superposition of 2 waves defined earlier) and Kirchhoff's laws at the external terminals or by following more formal procedure from [4], we can define S-parameters or S-matrix that relates the incident and reflected waves for such segment as follows:

$$S(f,l) = \begin{bmatrix} \left(Z_c^2 - Z_0^2\right)/D & 2 \cdot Z_c \cdot Z_0 \cdot csh(\Gamma \cdot l)/D \\ 2 \cdot Z_c \cdot Z_0 \cdot csh(\Gamma \cdot l)/D & \left(Z_c^2 - Z_0^2\right)/D \end{bmatrix}$$

$$D = Z_c^2 + Z_0^2 + 2 \cdot Z_c \cdot Z_0 \cdot cth(\Gamma \cdot l)$$

The reflection (S11 and S22) and transmission (S12 and S21) can be expressed separately as follows:

$$S_{1,1} = S_{2,2} = \left(Z_c^2 - Z_0^2\right) / \left(Z_c^2 + Z_0^2 + 2 \cdot Z_c \cdot Z_0 \cdot cth(\Gamma \cdot l)\right)$$

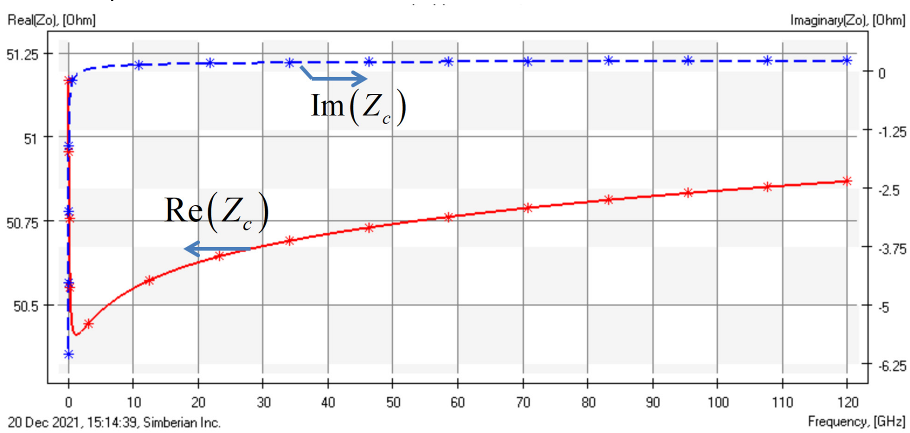
$$S_{1,2} = S_{2,1} = 2 \cdot Z_c \cdot Z_0 \cdot csh(\Gamma \cdot l) / \left(Z_c^2 + Z_0^2 + 2 \cdot Z_c \cdot Z_0 \cdot cth(\Gamma \cdot l)\right)$$

Note that the transmission parameters include the effects of the absorption and reflections – there are no approximations in these expressions. This is very universal definition of the reflection and transmission – can be used for simple experiments with transmission line properties or as rigorous modelling of a segment. It depends on the definition of characteristic impedance and propagation constant you put in there. The rest is pure trigonometry! You can start with a frequency-independent capacitance and inductance per unit length or use some more complicated expressions for the characteristic impedance and propagation constant such as used in [5]. For simple experiments, the propagation constant can be defined analytically with formulas or simply with phase delay or propagation velocity for ideal lines (see Appendix). This is very simple and important tool for all kinds of experiments in the frequency domain with real transmission lines. It includes all reflections in in time domain (if model bandwidth is properly defined [1])! Though, use of frequency domain response for time-domain analysis is not as easy [4]. Simbeor software is used here for all frequency and time domain analyses – it makes our investigation much easier.

Now, what useful information can be derived from such a simple trigonometric model? Let's begin from very simple case of the termination or normalization impedance equal to the characteristic impedance *Zo=Zc* - the reflection parameter is zero in this case as we can see from the formula! S-matrix in this case is particularly simple and defined as follows (generalized modal S-parameters):

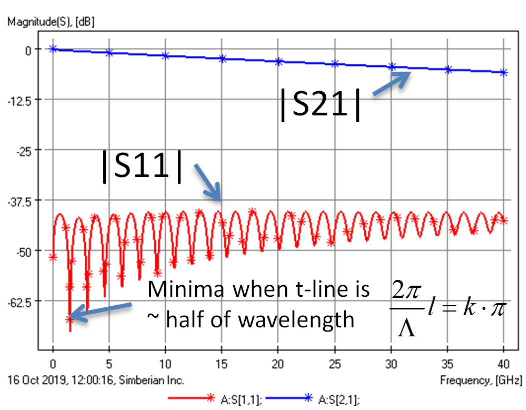
$$S(f,l)=\begin{bmatrix} 0 & \exp(-\Gamma\cdot l) \\ \exp(-\Gamma\cdot l) & 0 \end{bmatrix}$$

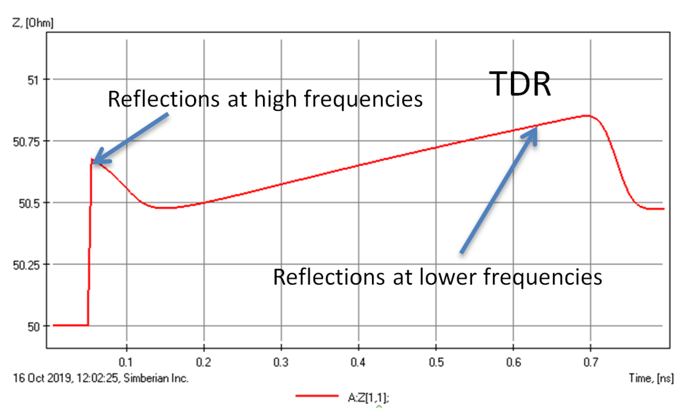
Only the transmission parameters and no reflections! - This should be the Holly Grail of the interconnect design – signal is travelling strictly in one direction. Though, the signal may still not get through because of the transmission parameter depends on the absorption and dispersion in Gamma discussed earlier in [2]. Considering the zero reflection condition, why we do not do it like that always? First, the characteristic impedance is complex for lossy lines – it has real and imaginary parts. The zero-reflection termination is not just a resistor – it should be frequency-dependent. But this is not the show stopper – the real part of the characteristic impedance does not change much at all important frequencies and the imaginary part is much smaller than the real part, as we can see from this plot (typical PCB case):



So, at least theoretically, we should be able to get very close to the non-reflective case. Practically there are more factors that do not allow it – the manufacturing variations and discontinuities such as pads and viaholes are the most important ones.

Now, armed with the theory, let's investigate a simple 5 cm strip line segment with characteristic impedance about 50.4 Ohm at 1 GHz (changing with frequency as shown above) on FR408 simulated as Wideband Debye with Dk=3.8, LT=0.0117 @ 1 GHz, copper with RR=1.2, Causal Hammerstad Roughness Model: SR=0.4, RF=2. The problem is as realistic as it can be and the only simplification is the absence of the discontinuities. The transmission line segment has the following response in frequency and time domains (computed with Simbeor software):

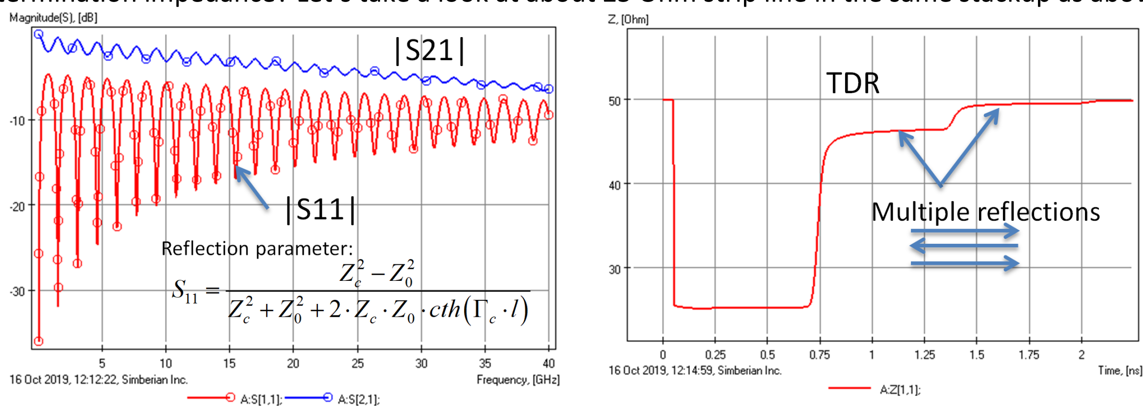




Both ends of the transmission line segment are terminated by 50 Ohm (exactly). Magnitudes of the reflection |S11| and transmission |S21| parameters are shown on the left plot and corresponding TDR on the right plot (reflection from 20 ps step response in Ohm). S-parameters are shown in dB

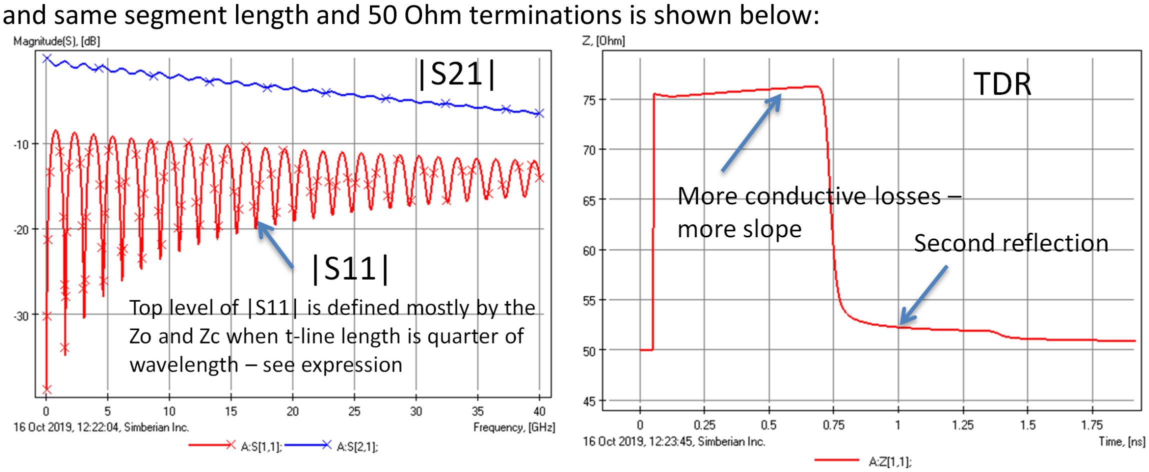
(20log(|S11|) and 20log(|S21|)). First, we can observe that the reflection is not zero, but very low – below -37.5 dB (only about 13 mV is reflected with 1 V excitation – it is as good as is usually not possible). As a consequence, the transmission parameter magnitude is smooth and is defined mostly by the absorption by dielectric and conductors. Notice that the reflection parameter has some minima and maxima. The first maximum is at frequency where segment length is about equal to a quarter of wavelength in transmission line, defined by Gamma (see Appendix) and repeating every half of wavelength. The first minimum is at about half of wavelength is also repeated every half of the wavelength (explained below). Value of the reflection at one frequency point may be misleading. Considering the TDR, we can see that it shows some variations consistent with the variations of characteristic impedance – see more on that at [6].

What if the characteristic impedance of the transmission line is significantly different from the termination impedance? Let's take a look at about 25 Ohm strip line in the same stackup as above:



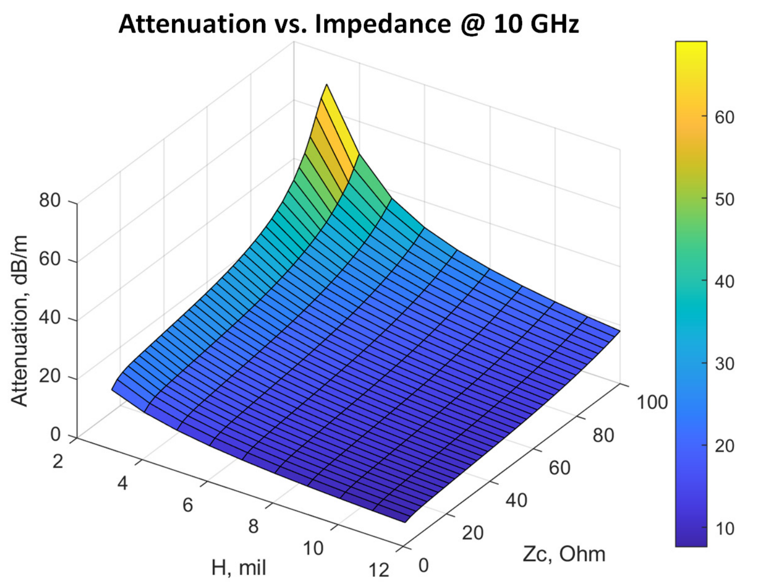
Magnitudes of the transmission (insertion loss) and reflection in dB are shown on the left plot and TDR on the right. The reflection went up considerably that means more signal energy is reflected. As the result, the transmission or insertion losses went down at some frequencies – less signal energy is transmitted. The insertion loss now is wavy and repeats the reflection pattern – maxima in the reflection are the minima in the insertion losses. The signal energy here is either reflected or absorbed. The left plot also has the expression for the reflection parameter – the hyperbolic tangent in the denominator explains the minima and maxima - it is trigonometry! Though with the complex numbers. S-parameters are used directly to compute the TDR that shows some multiple reflections from the ends of the segment in this case.

Another case with considerably larger characteristic impedance about 75 Ohm (cannot be exact) and same segment length and 50 Ohm terminations is shown below:

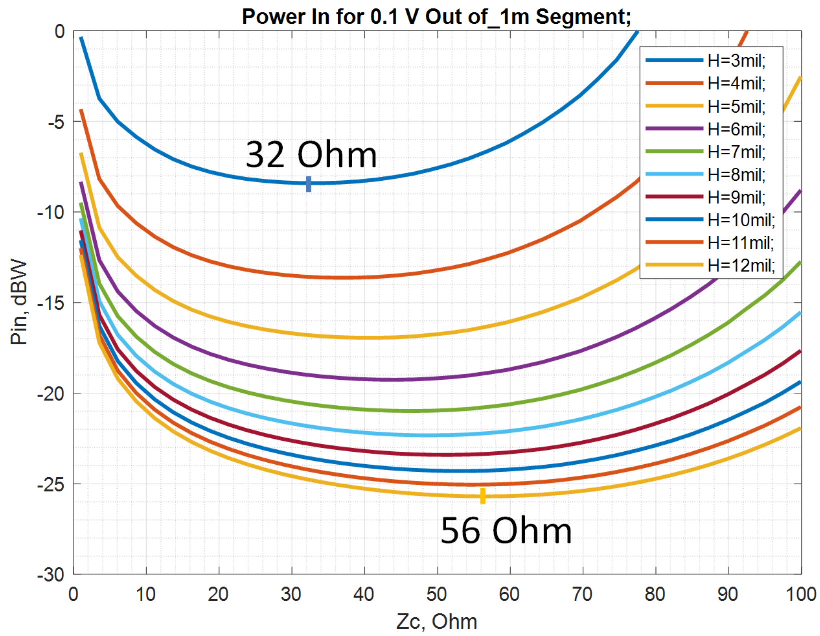


On S-parameter plot it looks very similar to the previous case. Though, it has more conductive losses and the TDR goes up, instead of down, and shows more resistance (slope up) in the narrow transmission line as expected. In both "reflective" cases only one or two reflections are significant – it disappears quickly due to the absorption losses (losses are our friend in such reflective cases).

If you are wondering why characteristic impedance equal to 50 Ohm is usually selected for single-ended and 100 Ohm for differential PCB or packaging interconnects, you are not alone. It can only be explained by the historic reasons and convention for the component terminators. In fact, there are no reasons to stick with this number. As the story goes**,** 50 Ohm was the tradeoff impedance of an air-filled coaxial transmission line between the maximal transmitted power and minimal losses [7]. Indeed, a coaxial line always has a minimum in losses vs impedance. Though it is dependent on the dielectric fill, but it happen to be close to 50 Ohm for coaxial lines filled with PFTE-type dielectric with Dk close to 2 (can be easily verified, [7]). **As we know, striplines are the descendants of the coaxial transmission lines, but the stripline losses do not have minimum on the loss vs. impedance function.** Here is the attenuation in dB/m for a strip line modeled with Dk=3.5, LT = 0.002 @ 1.0e9, Huray-Braken roughness model: SR = 0.1 um, RF = 9 as a function of dielectric thickness and characteristic impedance at 10 GHz (computed with Simbeor SDK for Matlab):

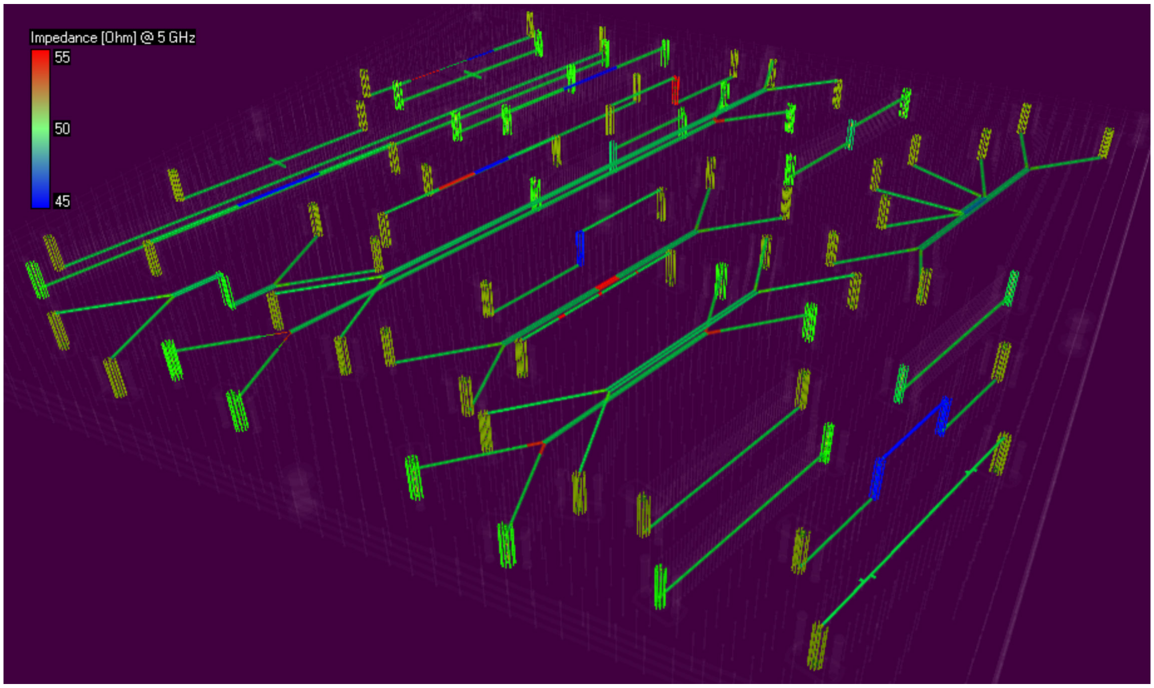


The attenuation is simply smaller for the smaller characteristic impedance (Zc axis) as well as for the thicker dielectrics (H axis). As was shown in [2] the conductor losses dominate in striplines with very low loss dielectrics. It means that the cross-sections with more metal and lower impedance have smaller losses in general. Though, the single mode propagation condition and layout density may put additional bounds on the increase of the cross-section size and on the lowest impedance as well. So, is the lower impedance always better? Not really, if our goal is to minimize the power absorbed by the interconnects and terminators. For instance, if we need 0.1 V signal at the receiver and compute power required at the transmitter side (Pin=20log(Vout)-10log(|Zc|)+Att_dB*Length, dBW), we will see some minima (same example as above at 10 GHz, computed with Simbeor SDK for Matlab):



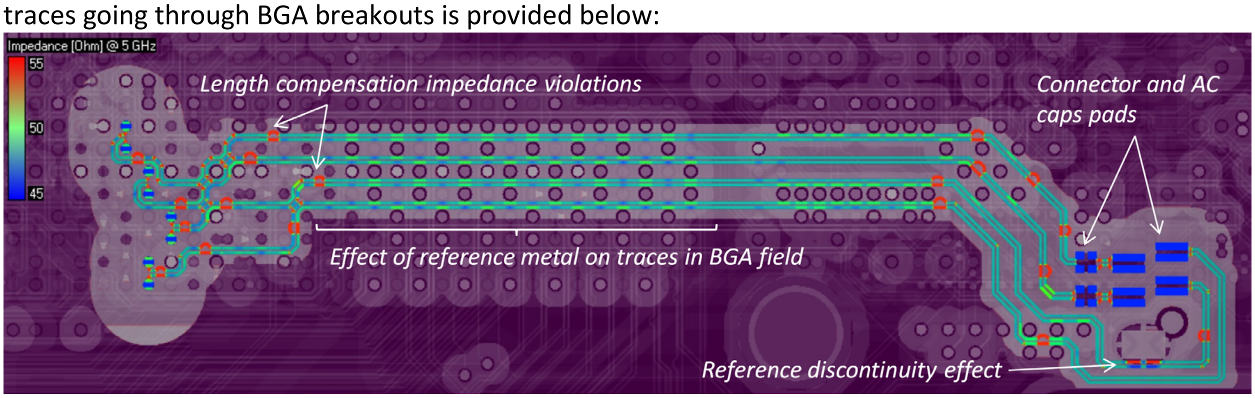
The minimal power depends on the geometry (dielectric thickness H above and below trace and trace width adjusted to have impedance value on the horizontal axis) and also on length (graphs above are for 1 m segment). Smaller impedance should be considered for thinner dielectric layers. Strip widths in this example are set to have the impedance shown on the horizontal axis (Simbeor SDK used for computations). Terminations in this case were set equal to magnitude of the characteristic impedance at 10 GHz (no reflections). **As we can see, the lower characteristic impedance is not always better and may be optimized for a particular system.**

Finally, the constant impedance from component to component should be the design goal, but it is usually violated in practical cases. The single-ended or differential traces are the open waveguiding structures composed of traces and also reference conductors. Though, almost all layout tools are not aware of that. Thus, before any type of interconnect analysis, the impedance continuity should be verified first with a validated field solver. Here is an example of such impedance verification for CMP-28 validation platform from Wild River Technology (https://www.wildrivertech.com/) in Simbeor 2022.02:



Green color is used for objects with the impedance close to the target impedance (50 Ohm single-ended or 100 Ohm differential). Objects with the impedance below the target are blue and with higher impedances are red. This is well designed board with small number of intentional impedance violations in some structures. Also it comes from Wild River Technology with the measurements up to 50 GHz for validation purpose. Simbeor evaluates the input impedance of the vias with fast EM models of multi-vias (signal + stitching vias) and impedance of traces with Simbeor SFS 2D field solver at the Nyquist frequency.

Another example of how the reference conductors can change the impedance of traces on a design with traces going through BGA breakouts is provided below:



Here Simbeor evaluated effect of the cut-outs and reference pads on the impedance – those cannot be avoided. We can see that the impedance of connector and AC coupling pads is below the target and the impedance of the length compensation sections is above the target (layout mistake). The discontinuities in the reference conductors also create impedance violations (another layout mistake). Though, most of those violations may not kill the signal and are important at relatively high data rates.

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Appendix: Other useful transmission line modal parameters derived from the complex propagation constant (Gamma) and useful for understanding of transmission line behavior (omega is the radial frequency [rad/s]):

$$\Gamma_n(\omega) = \sqrt{z_{n,n}(\omega)\cdot y_{n,n}(\omega)} = \alpha_n + i\beta_n$$

$\alpha = \mathrm{Re}(\Gamma)$ attenuation constant [Np/m]

$\alpha_{dB} = \dfrac{20\cdot\alpha}{\ln(10)} \approx 8.686\cdot\alpha$ attenuation constant [dB/m]

$\beta = \mathrm{Im}(\Gamma)$ phase constant [rad/m]

$\Lambda = \dfrac{2\pi}{\beta}$ wavelength [m]

$\varepsilon_{eff} = \mathrm{Re}\left[-\left(\dfrac{c\cdot\Gamma}{\omega}\right)^2\right]$ effective dielectric constant

$p = \dfrac{c}{\upsilon_p} = \dfrac{c\cdot\beta}{\omega}$ slow-down factor, *c* is the speed of electromagnetic waves in vacuum

$\upsilon_p = \dfrac{\omega}{\beta}$ phase velocity [m/sec]

$\tau_p = \dfrac{\beta}{\omega}$ phase delay [sec/m]

$\upsilon_g = \dfrac{\partial\omega}{\partial\beta}$ group velocity [m/sec]

$\tau_g = \dfrac{\partial\beta}{\partial\omega}$ group delay [sec/m]